Non-well-founded Probabilities within Unconventional Computing

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Abstract

In the paper we consider an opportunity of physical measurement based on the non-Kolmogorovian probabilistic model with non-well-founded probabilities for $p$-adic and adelic theoretical physics. Surprisingly such a departure from the conventional probability model generates departure from conventional set theory, namely, toward non-well-founded set theory and additionally departure from the conventional computing based on the Church-Turing thesis toward unconventional computing based on coinduction.

A non-well-founded (non-WF) set theory belongs to axiomatic set theories that violate the rule of WF-ness and, as an example, allow sets to contain themselves: $X \in X$. In non-WF set theories, the foundation axiom of Zermelo-Fraenkel set theory is replaced by axioms implying its negation. The negation of the axiom of foundation causes that there are objects that cannot be defined by Noetherian induction. This means that there exists a non-empty subset of $X$ that has no minimal element with respect to a relation $R$. The latter is called a non-WF relation. We remember that there are no least upper bounds for non-Archimedean numbers in the general case (notice that the completeness axiom is equivalent to the principle of continuous induction). This means that those sets of upper bounds have no least (minimal) element. Therefore we cannot use Noetherian induction for the set of infinitesimals or for the set of infinitely large integers (notice that sets $^*\mathbb{Z}\setminus\mathbb{Z}$ and $\mathbb{Z}_p\setminus\mathbb{Z}$ of infinitely large integers have a different meaning). Instead of Noetherian induction, we can use coinduction as the dual notion applied to non-WF objects. In this case, a binary relation, $R$, is non-WF on $X$ if and only if every non-empty subset of $X$ has a maximal element with respect to $R$ (this definition is one of the possible formulation of anti-foundation axiom).

The conventional approach to the physical measurement for statistical phenomena uses classical (Kolmogorov’s) probability theory built in the language of WF mathematics of real numbers. It sets a framework of modern physics, taking into account that physical reality is regarded in modern science as reality of stable repetitive phenomena (phenomena that have probabilities, i.e. do not fluctuate in the standard real metric). However, we can assume another approach to the physical measurement for statistical phenomena that uses tools of non-WF mathematics, in particular uses the coinduction principle and non-WF probability theory on non-Archimedean structures.

If in the real topology statistical stabilization is absent, then it is not possible to obtain any physical constants in the language of ordinary probability theory. But these constants could exist and be, for example, $p$-adic numbers. If a collective has not only a real probability distribution but an entire spectrum of other distributions, then, besides real constants corresponding to physical properties of the investigated objects, we obtain an entire spectrum of new constants corresponding to physical properties that were hidden from the real statistics. In accordance with assuming that reality is non-WF, experimental results may be analyzed not only in the field of real numbers but also in $p$-adic fields (or more generally, on streams). Some recent applications of unconventional (non-WF) approach to experimental data using distribution on $p$-adic fields are as follows: a) in cognitive science and neurophysiology; b) logical foundation of $p$-adic probability; c) $p$-adic cosmology and quantum physics.

Keywords. non-well-founded probabilities, $p$-adic probabilities, $p$-adic quantum mechanics, coinduction.