Imprecision in a Timber Asset Sale Model: Motivation, Specification, and Behavioral Implications

Mark J. Ducey
University of New Hampshire
mjducey@cisunix.unh.edu

Abstract

Timber management involves making long-term investment decisions. However, timber prices are characterized by interannual volatility, and the future costs and revenues of management depend on a changing social, technological, and environmental context. Timber management with fluctuating prices is commonly analyzed using an asset pricing model that can be solved as a recursive dynamic program. I reformulate the model in terms of previsions for gambles, and introduce imprecision in both future prices and discount rates. Imprecision in prices and intertemporal preferences leads to imprecise buying and selling prices for timber and for timberland. The results have behavioral implications which may assist in understanding individual landowners and timber markets.

Keywords. timber management, investment analysis, intertemporal preferences, upper and lower previsions.

1 Introduction and Context

Managing forests for sustainable timber production involves making investment decisions over time scales that are quite long, relative to many conventional investments such as stocks or commercial real estate. Even when viewed from a strictly financial standpoint, timber management must address several sources of uncertainty. Biological yields are imperfectly predicted by the best available models and data. Price per unit yield is usually modeled as the residual after management, marketing, and manufacturing costs are deducted from the value of end products. However, management costs change as labor markets, technology, regulation and social pressures transform methods of performing even the simplest tasks, such as felling timber and transporting it to a ready buyer. Perhaps even more striking is the impact of technology and changing social demands on the end products and markets themselves.

As an example, consider the harvesting of, and markets for, eastern white pine in New England. Fundamental changes in every aspect of this sector of the forest products industry have occurred over the timeline of investment in a single rotation. Logging using a crosscut saw and draft horses or oxen, has been replaced by equipment that did not exist (chain saws and cable skidders) when currently mature trees were first planted and tended. Chain saws and cable skidders are themselves being replaced, with significant impacts on the cost of harvesting. Marketing and processing of logs has changed from a small-scale, local use economy based on small operators with fixed or portable mills, to a regional, national, and international trade in raw materials supplying large, capital-intensive processing facilities. The end products themselves have changed: once-ubiquitous pine boxes and crates have been replaced by corrugated boxes, boards for structural use have been replaced by plywood and newer engineered materials, and specialty markets have been invaded by inexpensive plastics. As a consequence, white pine has declined from over 50% of the U.S. softwood supply to less than 5% [15]. All these changes directly impact the price paid to landowners for standing timber, and hence affect the desirability of forest land and forest management as investments.

While the transformation of management and markets within a single rotation of eastern white pine may be predictable in hindsight, prediction in advance requires envisioning shifts in products and technologies to materials and methods that may not even exist at the time an investment decision is made. Furthermore, the ethical imperative for sustainable management requires that we consider management consequences over multiple future rotations. Viewed from this perspective, forestry investment analyses that treat future prices for timber as known exactly to the decisionmaker, or as following a stochastic process the characteristics of which are known exactly to the decisionmaker, seem intuitively unsatisfactory, especially if they conceal underlying imprecision in knowledge about future conditions. Yet existing approaches to forestry investment analysis take one of these two paths. By contrast, forestry as an enterprise may prove fertile ground for applications of imprecise probability and related methods, to the degree that they can model ignorance about the future and equivocal preferences [7, 8].

Imprecise models for environmental decisions have been introduced by Chevé and Congar [6], and imprecision in asset pricing has been discussed by Epstein and Wang [9]. In this paper, I reexamine a timber asset sale model that typifies the analyses used to determine selling prices for timber, as well as purchase or sale prices for timberland, in a market characterized by price fluctuations [5]. The incorporation of imprecise probabilities in the model leads to imprecise values for selling prices for timber, as well as imprecise values for bare timberland. The results have behavioral and policy implications, which are also discussed.

2 Asset Sale Model

Consider, in its simplest form, the problem of deciding whether or not to harvest a stand of timber based purely on financial considerations. For simplicity, assume that the trees are all of the same age class t since planting. Assume also that we will cut all of the trees or none of the them, obtaining a yield Q_t which is an increasing function of t, based on the biological growth of the trees [1]. Assume furthermore that if we do harvest, we will use the land to grow another stand of trees, repeating the process *ad infinitum*; in other words, we intend to practice forestry in a sustained fashion. Establishing the new stand is straightforward but may require a cash expenditure for seedlings, labor, and so on.

If prices are constant, then the optimal age at which to harvest, also called the optimal rotation age, occurs when the value of additional growth exactly equals the opportunity cost of waiting for a future harvest. Specifically, in continuous time the Faustmann condition for harvesting [10] is

$$V\frac{\mathrm{d}Q}{\mathrm{d}t} = r(VQ + W) \qquad (1)$$

[1] In the simplest case, Q_t is measured in physical units of volume, such as cubic meters or board feet. However, there is usually a predictable pattern of change in grade and quality – and therefore price per unit volume – as a stand ages. A more general measure is that of *equivalent yield*, or yield normalized to units of comparable price.

where V is the price per unit of Q, W is the value of bare land, and $r > 0$ is an interest or discount rate. The discount rate is chosen based on the expected return of investments of comparable risk, and serves to calculate the cost of deferring income. The value of a future return, discounted to the present, is called its net present value. The value of bare land is taken as the net present value of an infinite series of similar rotations, starting just after harvest and including any costs associated with reforestation.

When prices fluctuate, the owner of a stand of timber faces a slightly different problem. Given a known price today V_t and unknown prices in the future $V_{t+1}..V_\infty$, should the owner sell, or wait for a higher price? The formulation in eq. 1 is no longer appropriate; a model incorporating stochastic prices is needed. Asset sale models to contend with stochastic prices were originally introduced by Karlin [17], and subsequently adapted to forestry applications by Brazee and Mendelsohn [5]. Intuitively, if V_t is low, the owner should defer harvest, while if V_t is high, the owner should wish to "cash in". While the stand is young and growing quickly, only a very high price should be sufficient inducement to sell. However, as the stand ages, a lower and lower price should be sufficient. Denote this *reservation price* as X_t.

The optimal decision rule for a risk-neutral decisionmaker is similar to the principle outlined in eq. 1. Specifically, the timber owner should sell if and only if the revenue from selling would equal or exceed the expected net present value of future income that would accrue from waiting [5]. Mathematically,

$$[X_t Q_t + E(W)] = \sum_{j=t+1}^{T} U_{t+1,j-1} e^{-r(j-t)} R_j \qquad (2)$$

where

$$R_j = P_j[E(V_j | V_j \geq X_j) Q_j + E(W)] \qquad (3)$$

The value T represents a maximum harvest age that will be considered (*i.e.* all stands will be harvested at $t = T$). Technically, T should be ∞, but due to the effect of discounting, the solution is numerically indistinguishable if T is taken as several times the Faustmann rotation age, and it simplifies solution of eq. 2 considerably if T is finite. Bare land value is replaced by its expectation, since W is now a random variable, as described below. Assuming that V_t can be described by some unique probability density function $f_t(V)$, P_t is the probability that the stand will be harvested at age t, *i.e.*

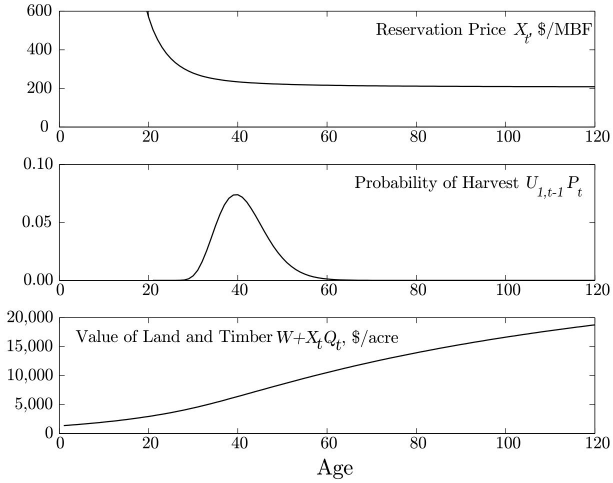

Figure 1: Solution of the precise model for eastern white pine.

$$P_t = \int_{X_t}^{\infty} f_t(V)\mathrm{d}V \tag{4}$$

and

$$E[V_t|V_t \geq X_t] = \frac{\int_{X_t}^{\infty} V f_t(V)\mathrm{d}V}{P_t} \tag{5}$$

If the V_t are independently and identically distributed with p.d.f. f, with $f_t(V) = f(V)$, then $U_{t,j}$, which represents the probability that the stand is *not* harvested between ages t and j (inclusive) is simply

$$U_t, j = \prod_{k=t}^{j}(1 - P_k) \tag{6}$$

As noted above, W is now a random variable, because the age at rotation is also a random variable. Its expectation can be calculated as

$$E(W) = -C + \sum_{t=1}^{T} U_{1,t-1} R_t e^{-rt} \tag{7}$$

where C is the cost of reforestation. If Q_t is known, r is uniquely defined, and f is likewise uniquely defined, then eqs. 2 and 7 can be solved recursively from Z to 1 as a dynamic program. The solution must be performed iteratively to obtain $E(W)$, but convergence is usually good [5].

Figure 1 shows aspects of the solution of the model for eastern white pine growing on a moderate site in the northeastern U.S. The interest rate is taken as $r = 0.04$. f is taken as i.i.d. Normal, with mean \$200/MBF and a standard deviation of \$30/MBF. Q_t is calculated as

$$Q_t = e^{5.0981 + \frac{-80.9557}{t}} \tag{8}$$

where Q_t is in thousand board feet per acre (MBF/acre), based on published yield tables for eastern white pine [19]. Since eastern white pine is usually regenerated naturally, rather than by planting, we let $C = 0$. Under these conditions, the expected value of bare land is \$1,328. The reservation price X_t decays rapidly toward its asymptotic value near \$207/MBF. The expected age at harvest is 41.4 years, slightly less than the Faustmann rotation age of 45 years. 90% of all stands are harvested between ages 33 and 50, inclusive. The value of $X_tQ_t + E(W)$ serves as the basis for the reservation price for the land *and* timber. Since the future yield and discount rate are precise, and since the distribution of future prices is precise, from a dogmatic Bayesian standpoint these values have a precise normative interpretation. For example, the value of land and timber at age 20 is \$2,956. Any offer greater than \$2,956 per acre – say, \$2,957 – should be sufficient inducement for the owner to sell a 20-year-old forest outright, since accepting the offer leads to a gain in expected utility; at an offering price of \$2,955 per acre, one should be willing to buy similar timberland without trepidation. From a pragmatic standpoint, one would be unlikely to interpret the model so strictly, especially given the difficulty in predicting future market conditions. By implication, either the pragmatist is irrational, or the model does not deal with underlying imprecision.

Discussion and elaboration of this basic model and similar models for timber investment decisions are found in [5, 12, 13]. In the following sections, I explore what happens when imprecision is introduced into two of the key variables. First, I define an imprecise formulation of the problem. Then, I explore what happens when f is replaced by an imprecisely specified probability density function. Next, I address modeling subjective uncertainty about an appropriate discount rate using imprecise probabilities. Both formulations lead to imprecise values for X_t, but with different characteristic impacts. The practical ramifications of these impacts are discussed in the succeeding section.

3 Imprecise Asset Sale Model

Solution of eq. 2 is straightforward when $\mathbf{Q}$, C, r, and f are known. From a traditional Bayesian standpoint, solution is also reasonably straightforward if any uncertainty about $\mathbf{Q}$, C, r, or f can be represented by appropriate and unique probability distributions (or, in the case of f, hyper-distributions), since all of the relevant expectations are uniquely defined. In other words, extension to a traditional Bayesian approach is fairly obvious, though no such extension of this specific model has appeared in the forestry literature.

However, based on the general context given in Section 1, and as discussed below in Sections 4 and 5, it may not be reasonable to expect that a decisionmaker will be able to assign a precise probability density function to $\mathbf{Q}$, C, r, and f or their relevant parameters. In this case, the model must contend with imprecision, and this requires some reformulation. For simplicity, we restrict our attention here to imprecise specification of r and f, though extension to imprecision in $\mathbf{Q}$ and C is possible [2].

We approach the formulation of an imprecise model by recasting the precise model in terms of prices for gambles [25]. Consider the form of eq. 2 at $t = T$, when $X_T = -\infty$. When f is a precise probability for V_T, we receive a gamble that has a precise value G_T:

$$G_T = \mathrm{E}(V_T)Q_T + \mathrm{E}(W) = \mathrm{E}(V_T)Q_T - C + e^{-r}G_1 \quad (9)$$

Now, suppose $f(\mathbf{V})$ is imprecise. Then G_T will also be imprecise, and we can represent that imprecision using upper and lower previsions. Specifically,

$$\begin{aligned} \overline{G}_T &= \overline{V}_T Q_T + \overline{W} \\ \underline{G}_T &= \underline{V}_T Q_T + \underline{W} \end{aligned} \quad (10)$$

where $\overline{G}_T$ and $\underline{G}_T$ are the upper and lower previsions (selling and buying prices) for the gamble at time T, and $\overline{V}_T$ and $\underline{V}_T$ are upper and lower previsions for V. W will also be imprecise, and defined by

$$\begin{aligned} \overline{W} &= -C + \overline{\mathrm{E}}_{\mathbf{V},r}(e^{-r}G_1) \\ \underline{W} &= -C + \underline{\mathrm{E}}_{\mathbf{V},r}(e^{-r}G_1) \end{aligned} \quad (11)$$

What is the value of owning the forestland gamble at time $T-1$, just before observing price V_{T-1}? In the precise case, we may discern two possible outcomes. Given a reservation price X_{T-1}, we receive

$$\begin{aligned} & V^X_{T-1}Q_{T-1} + \mathrm{E}(W) && \text{if } V_{T-1} \geq X_{T-1} \\ & e^{-r}G_T && \text{otherwise} \end{aligned} \quad (12)$$

where

$$V^X_{T-1} = \mathrm{E}(V_{T-1} | V_{T-1} \geq X_{T-1}) \quad (13)$$

and the optimal choice of X_{T-1}, in the sense that it maximizes the discounted net present value of the gamble at time $T-1$, satisfies

$$X_{T-1} = \frac{e^{-r}G_T - \mathrm{E}(W)}{Q_{T-1}} \quad (14)$$

From a dogmatic, conventional viewpoint, this value is normative: it is the only precise reservation price that leads to coherent choice. The reservation price X_{T-1} is the value of the difference of two gambles: the discounted future gamble G_T, and the bare land gamble taken immediately, per unit yield Q_{T-1}. The value of the gamble at time $T-1$ is

$$\begin{aligned} G_{T-1} \quad = \quad & P_{T-1}\left[V^X_{T-1}Q_{T-1} + \mathrm{E}(W)\right] \\ & + (1 - P_{T-1})\, e^{-r}G_T \end{aligned} \quad (15)$$

It is immediately clear that eqs. 12 through 15 generalize from times $T-1$ and T to times $t-1$ and t, and in the precise case provide a recurrence relationship for calculating $\mathbf{X}$ and $\mathbf{G}$.

Now, if f, r, W and G_t are imprecise, then we must admit the reservation value X_{t-1} should also be imprecise. Because X_{t-1} represents a price for a gamble, its imprecision could be modeled by coherent upper and lower previsions, and it ideally would be calculable by the upper and lower previsions of its component gambles. Unfortunately, $e^{-r}G_T$ and $\mathrm{E}(W)$ are not independent gambles, because both depend on the underlying imprecisely specified random variables r and V_T. In general,

$$\begin{aligned} \overline{X}_{t-1} &= \overline{\mathrm{E}}_{\mathbf{V},r} \frac{C + e^{-r}(G_t - G_1)}{Q_{t-1}} \\ \underline{X}_{t-1} &= \underline{\mathrm{E}}_{\mathbf{V},r} \frac{C + e^{-r}(G_t - G_1)}{Q_{t-1}} \end{aligned} \quad (16)$$

Given $\overline{X}_{t-1}$ and $\underline{X}_{t-1}$, the upper prevision for the gamble at $T-1$ is

$$\begin{aligned} \overline{G}_{t-1} = & \max_{X_{t-1}} \overline{\mathrm{E}}_{\mathbf{V},r}\{e^{-r}G_t \\ & + P_{t-1}[V^X_{t-1}Q_{t-1} - e^{-r}(G_t - G_1) - C]\} \end{aligned} \quad (17)$$

[2] Extension to C is straightforward, but uninteresting. Extension to $\mathbf{Q}$ is complicated by the fact that for some t in the current rotation, Q_t may be known exactly, or described probabilistically by sample data. Useful extension of the model requires incorporating this knowledge into assessments of future Q_t in the current rotation, as well as $\mathbf{Q}$ for future rotations.

where maximization is over $\underline{X}_{t-1} \leq X_{t-1} \leq \overline{X}_{t-1}$. The lower prevision for the gamble at $t-1$ is defined similarly, with min replacing max and lower replacing upper expectation. Note that $\underline{G}_{t-1}$ often will not correspond with $X_{t-1} = \underline{X}_{t-1}$; the worst case is that future $\mathbf{V}$ and r are unfavorable, and that the decisionmaker acts suboptimally, rather than optimally, due to imperfect information.

The feasibility of solving eqs. 16 and 17 for $t = 1...T$ may depend on how imprecision in $f(\mathbf{V})$ and r is specified. However, for simple specifications analogous to those commonly employed in the precise case, solution is straightforward. We consider examples of specifications and their corresponding solutions next.

4 Uncertainty about Future Prices

Although alternative models for future prices might be more realistic [18, 24], the most commonly employed model for future timber prices is an i.i.d. normal distribution [5, 12, 13], and it serves well to illustrate the role of price fluctuation in returns on timber investments. However, future timber prices in the precise models devloped so far assume the parameters μ and σ of the distribution are precise and known to the decisionmaker. A traditional Bayesian approach would use subjective information and/or data on previous prices to construct precise hyperdistributions for μ and σ, leading to a precise (but possibly non-normal) distribution for V_t. In practice, μ and σ for some future price V_t are not known, and construction of a precise hyperdistribution may not be possible or desirable. Consideration of the sources of imprecision discussed by Walley [25, Section 5.2], indicates several reasons this may be so. Specifically,

- In a Bayesian analysis, assessment of precise hyperpriors may not be possible because of inadequate time or procedures [3], or the discomfort or unfamiliarity of the decisionmaker with probabilistic assessments.

- Available data on previous prices may not be relevant to the timber resource at hand. Although many sources of past price data are usually available, these typically represent price data averaged over many sales and a large geographic area, and methods of reporting not always consistent [20]. Distance to facilities, road accessibility, and terrain all affect prices for timber on a particular property, but these effects may be difficult to assess.

- Available data on previous prices may not be relevant to future prices. Prices for standing timber are usually considered as the residual value after cutting, hauling, processing, and marketing costs are deducted from the price of a finished product. Technologies and costs of all these steps, as well as the definitions themselves of finished products, are constantly and in some cases rapidly evolving.

- The analysis itself may be viewed as approximate.

Instead of a distribution $f(\mathbf{V})$ characterized by precise μ and σ, an attractive imprecise specification is $f(\mathbf{V}) \in \mathcal{F} : \underline{\mu} \leq \mu \leq \overline{\mu}, \underline{\sigma} \leq \sigma \leq \overline{\sigma}, \forall t$ [3]. It provides an imprecise specification leading to coherent previsions, but is also uncomplicated and lends itself to interpretation by forest managers, who are not usually statisticians or economists. As such, it represents an intellectually tractable approximation to more complicated models involving, for example, upper and lower distributions for μ and σ. It also leads to a straightforward simplification of eqs. 16 and 17, allowing ready solution. Taking r as precise, and recalling that $\overline{X}_T = \underline{X}_T = -\infty$,

$$\begin{aligned} \overline{G}_T &= \overline{\mu} Q_T - C + e^{-r} \overline{G}_1 \\ \underline{G}_T &= \underline{\mu} Q_T - C + e^{-r} \underline{G}_1 \end{aligned} \tag{18}$$

$$\begin{aligned} \overline{X}_{t-1} &= \frac{C + e^{-r}(\overline{G}_t - \overline{G}_1)}{Q_{t-1}} \\ \underline{X}_{t-1} &= \frac{C + e^{-r}(\underline{G}_t - \underline{G}_1)}{Q_{t-1}} \end{aligned} \tag{19}$$

$$\begin{aligned} \overline{G}_{t-1} &= G(\overline{X}_{t-1}, \overline{\mu}, \overline{\sigma}, \overline{G}_t, \overline{G}_1) \\ \underline{G}_{t-1} &= G(\overline{X}_{t-1}, \underline{\mu}, \underline{\sigma}, \underline{G}_t, \underline{G}_1) \end{aligned} \tag{20}$$

Eqs. 18 through 20 can be solved recursively and iteratively for $\overline{\mathbf{G}}$, $\underline{\mathbf{G}}$, $\overline{\mathbf{X}}$ and $\underline{\mathbf{X}}$. Note, however, that the validity of this simplification depends on the assumption that f is a normal p.d.f. For other distributions, other relationships and solution methods may be required. In the normal case, provided suitable initial estimates are used for $\overline{G}_1$ and $\underline{G}_1$, convergence appears rapid in all tests to date.

As an example, Figure 2 shows aspects of the solution for eastern white pine, with the same yield function and interest rate as in Fig. 1. However, $f(\mathbf{V})$

[3] Letting $f(V_t)$ be any member of a bounded set of probability measures, independent of $f(V_{t' \neq t})$, allows modeling not only imprecision in the specification of $f(V_t)$ but also in the dependence within $\mathbf{V}$ and the stationarity of the generating process. However, the solution is not as straightforward, and its properties remain under investigation.

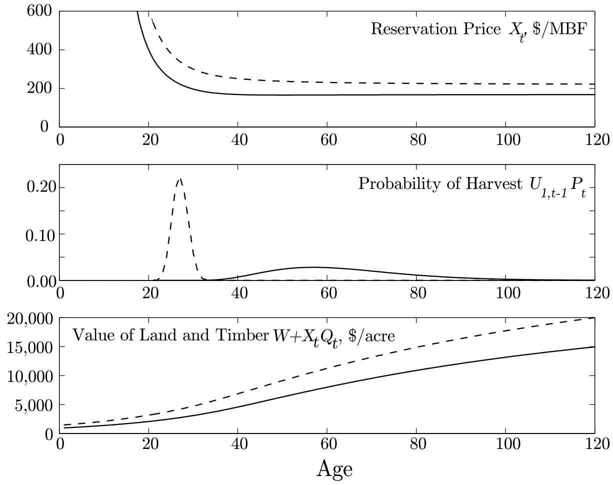

Figure 2: Solution of the model for eastern white pine, with imprecise future prices.

is now imprecise, with $\underline{\mu} = 190$, $\overline{\mu} = 210$, $\underline{\sigma} = 25$, and $\overline{\sigma} = 35$. The effects of imprecise (as opposed to precise probabilistic) knowledge of future prices on bare land value is dramatic: its lower prevision is only \$934, while its upper prevision is \$1,425. The effect on reservation price is readily apparent from the top panel of the figure. While both the upper and lower reservation price converge toward asymptotes, these now differ by \$52/MBF. As a result, harvesting behavior is indeterminate. The second panel depicts the two extreme compatible probability distributions for harvesting behavior, both of which occur when the landowner behaves suboptimally. The first possibility is that prices will be high and volatile ($\mu = \overline{\mu}$ and $\sigma = \overline{\sigma}$), but the landowner consistently sets a policy based on $\underline{\mathbf{X}}$, so harvesting is premature. The second occurs when prices are low and stable ($\mu = \underline{\mu}$ and $\sigma = \underline{\sigma}$), but the landowner sets prices based on $\overline{\mathbf{X}}$, expecting to hit a jackpot that never arrives. The lower prevision for rotation age is only 27.1 years, while the upper prevision is 64.1 years. As a result of both imprecision in price and indeterminacy of landowner behavior, the value of the land itself shows considerable imprecision. For example, the lower prevision for land and timber at 20 years of age is \$2,079, while the upper prevision is \$3,173. A market for land and timber that results in prices below \$2,079 for such land provides certain inducement to buy, while a market providing prices above \$3,173 provides certain inducement to sell. In between, the decision to hold or sell land is equivocal, and apparently minor factors may have a substantial impact.

5 Uncertainty about Appropriate Discount Rate

In the basic Faustmann model, the interest rate r represents the time cost of money, *i.e.* the cost of deferring rewards into the future. As Arrow [2] points out, discounting is required because present investments can make more capital available for investment or consumption in the future. Because of the inherently long time scales involved in forestry, forest investment decisions are quite sensitive to the choice of r. According to the conventional view, r should be uniquely defined, at least given sufficient data and analysis, though it may depend on the characteristics of the individual decisionmaker and the investment. However, considering to the most common methods of selecting r, it should not be surprising if imprecise specification of r may more accurately reflect the decisionmaker's preferences:

- The discount rate may be derived by direct elicitation, *e.g.* by asking the decisionmaker to compare the value of specified future returns at future dates to some fixed present return, and indicating preferences directly. This approach suffers all the drawbacks of finite elicitation, as discussed by Berger [3]. Imprecision will still arise if the decisionmaker finds the choice between some future return and the present return genuinely indeterminate.

- More commonly, the discount rate is set based on a commercial lending or borrowing rate available to the decisionmaker. However, over the long time scales inherent in forestry, such rates are unlikely to remain constant (whether taken as nominal rates, or real rates net of inflation), and prediction may be difficult.

- A further difficulty with commercial rates is that they are usually taken to represent *minimum* rates: they reflect the cost of money under low-risk conditions. Techniques are available to adjust risk-free discount rates for the presence of risk, but methodological difficulties remain [11, 18, 27]. Furthermore, the risks not included in a model such as the basic one presented here include fire, wind, ice, disease, and other damaging agents. By virtue of their rare nature, it is difficult to quantify their frequency and magnitude precisely even in the past, much less under threats of changing climate. Thus, quantifying an actual discount rate in the face of risk remains problematic.

- Alternatively, portfolio theory suggests that appropriate discount rates for forestry investments are less than commercial rates, since timberland performance is negatively correlated with market portfolio returns. Approaches to setting appropriate discount rates in this case depend on a data-demanding capital asset model, and are only appropriate for investors who already own a diversified portfolio, and for companies owned primarily by such investors [23, 27].

- In some analyses, especially those involving policy decisions and public works, a social rather than financial discount rate is used [14, 21]. Such discount rates purport to capture social values not reflected in conventional analyses. However, given the pluralistic nature of social valuation, representing social values with a single, precise discount rate or rate adjustment seems difficult [1].

- Some controversy remains over whether a single discount rate can be applied across all the investment durations contemplated in forestry [4, 21, 27], though Binkley [4] provides evidence that apparently lower long-term discount rates can be derived from variability in short-term intertemporal preferences.

In forestry investment analysis as well as pedagogical practice, it is common to conduct a sensitivity analysis to show how changing the discount rate dramatically affects net present values (and hence the relative desirability of different investment choices). The decisionmaker (or student) is then advised to choose a single discount rate in order to render the decision unequivocal. Such an approach is intellectually unsatisfying and potentially misleading; it leads to a precise determination of preferences between gambles when the underlying subjective preferences and/or objective data are equivocal.

As an alternative, suppose the discount rate is formulated imprecisely, to reflect an equivocal intertemporal preference. Such an equivocal preference may be motivated by imperfect data, elicitation, or modeling; or it may reflect genuine indeterminism on the part of the decisionmaker. Examining eqns 10, 16 and 17, we find that all depend on e^{-r}, the discounting factor for utility deferred over the immediate coming year. As Binkley [4] discusses, this factor may vary from year to year. Its influence on $\overline{X}_{t-1}$ and $\underline{X}_{t-1}$ is monotonic, but its direction depends on the sign of $G_t - G_1$ (in upper and lower expectation). Its influence on $\overline{G}_{t-1}$ and $\underline{G}_{t-1}$ is likewise monotonic with direction depending on the sign of $G_t - P_{t-1}(G_t - G_1)$ in upper and lower expectation. Now, G_t and G_1 depend on e^{-r}, but in a different year. So, if we specify upper and lower previsions for e^{-r}, we may perform the minimizations and maximizations in eqns 10, 16 and 17 independently for each year of the solution.

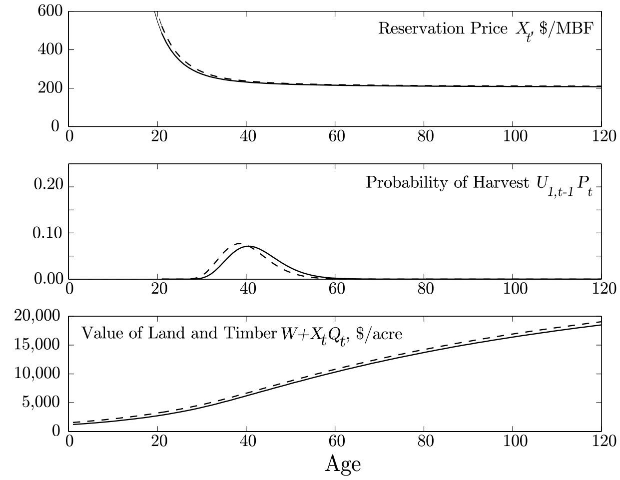

Figure 3: Solution of the model for eastern white pine, with imprecise discount rates.

An example is shown in Figure 3, with $\overline{e^{-r}} = 0.9632$ and $\underline{e^{-r}} = 0.9584$. These values correspond to precise values for r of 0.0375 and 0.0425, respectively. Future prices are taken as precisely specified with $\mu = 200, \sigma = 30$, as in Fig. 1. Although bare land values differ strongly (upper and lower previsions of \$1,513 and \$1,168, respectively), such low imprecision in discount rate appears to have little effect on harvesting behavior (upper and lower prevision for rotation age of 42.4 and 40.2 years). Values for land and timber are imprecise, though the degree of imprecision is not severe (upper and lower values at age 20 of \$3,204 and \$2,732). Based on this example, one might believe the impact of minor imprecision in discount rates is not important.

However, when combined with imprecision in future prices, the effects of imprecision in discount rates can be large. Figure 4 shows the effect of imprecise discount rates as in Fig. 3 with imprecise future prices as in Fig. 2. The upper and lower previsions for bare land value differ by a factor of more than 2 (\$1,625 and \$754, respectively). The upper and lower previsions for rotation age have spread farther apart than in Fig. 2, and are now 66.7 years and 25.3 years, respectively. As a result, the value of land and timber together is extremely imprecise, with upper and lower values at age 20 of \$3,440 and \$1,765, respectively. Because of the multiplicative nature of eqns 10, 16 and 17, an apparently unimportant source of imprecision becomes important when combined with other sources.

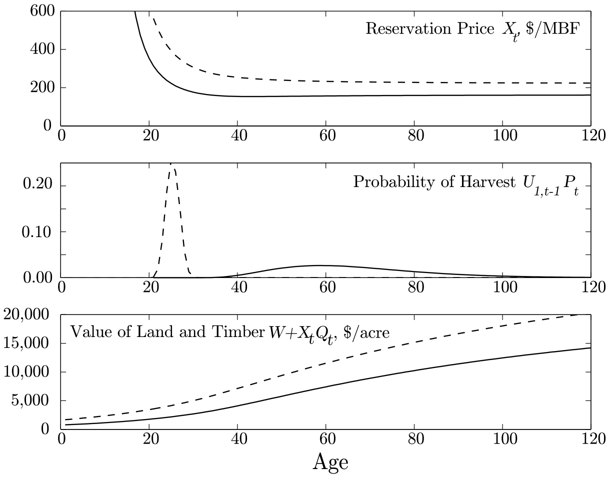

Figure 4: Solution of the model for eastern white pine, with imprecise future prices and discount rates.

6 Behavioral Implications and Conclusions

As the results of Sections 4 and 5 show, coherent landowner behavior with indeterminate preferences can arise from simple sources of imprecision. Furthermore, the resulting imprecision in reservation prices and land values can be large. These results help explain one counterintuitive result from Brazee and Mendelsohn [5]: that increasing price volatility benefits, rather than harms, landowners. It is certainly true that if a landowner can provide a precise specification of the distribution of future prices, the landowner can use that specification to set reservation prices which will increase the net present value of the land above that anticipated by the constant-price Faustmann model. However, when the specification is imprecise, the lower expected value of the land falls, and the landowner may not be able to state unequivocally that the land is worth more than the basic Faustmann model predicts.

Imprecise reservation prices may also help explain difficulty in predicting individual landowner harvesting behavior. Epstein and Wang [9] suggest that imprecise asset values may help explain excess volatility in markets. Anecdotally, the decision of many nonindustrial landowners to harvest is unplanned, sudden and does not reflect underlying changes in timber price. Such behavior is consistent with Keynes' notion of "animal spirits" in the economic decision process [9]. Imprecise land and timber prices, in conjunction with differences in goals and constraints between buyers and sellers, may also help explain rapid turnover in forest land ownership.

The model presented here assumes maximum imprecision in landowner behavior, consistent with coherence. Specifically, if $\overline{X}_t$ and $\underline{X}_t$ represent a future upper and lower reservation price, the landowner is fully noncommittal about whether harvest will occur if $\underline{X}_t \leq V_t \leq \overline{X}_t$. This lack of commitment increases the imprecision in $G_{t'}$ for $t' \leq t$. If the landowner is willing to commit to some policy for setting future X_t, the imprecision may be reduced. Of particular interest is a maximin strategy, in which X_t is chosen to maximize $\underline{G}_t$ $\forall t$. Significant increases in the lower expected value of land and timber may be realized. However, this must come at a cost to the upper expected value.

An important elaboration of the basic model presented here would be to incorporate long-term changes in expected prices. Many species and grades of timber have shown consistent real price increases over the past several decades [16, 18]. Protracted real price changes have substantial impact on the basic Faustmann model [26]; it is reasonable to conjecture that uncertain future price increases would magnify the role of imprecision in the timber asset sale model presented here.

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