Theories of Probability

Musings on a 30th Anniversary

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Taking It Personally: People

- Andrei N. Kolmogorov
- Jimmie (L.J.) Savage
- Max Black
- Patrick Suppes
A.N. Kolmogorov, June 1973
Outline of Reflections

- Pointing to the Subject of Probability
- Probabilistic Reasoning
- FoP Matters
- ToP
- Ruling Triumvirate
- Meaning
- Representation
- Axioms
- Closing
Pointing to the Subject
Origins-Egypt, c. 1500BCE

- Do we learn from history or select from it?
id facit exiguum clinamen principiorum
nec regione loci certa nec tempore certo.
Leibniz & [0,1]
Jacob Bernoulli (Basel)
Pointing to the Subject

- **Focal keywords:** random, chance, likelihood, probably, uncertainty, support, inference, prediction, decision

- **State of affairs:** partially known by, and with consequences for, a probabilistic reasoner (PR)

- PR need be no more subjective than is engineering
Common Realms of Discourse

- Natural--motion of air, water, magma, plasma, distribution of resources
- Physical--particle decay, galaxy pattern
- Biological--mutation, evolution,…
- Societal--unk unks, unintended conseq.
- Evidentiary--natural lang., legal, AI
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Methodology

- PR is more than probability theory—statistics, estimates, inferences, decision making,…
- How can we judge adequacy outside of the methodology itself?
- Is self-consistency sufficient or at best necessary? (Frequentist probability and LLNs)
- “But, it works!” Is a pragmatic stance sufficient or defeatist? How do we know what “works” or is possible?
Self-consistency

- From probability model $M$ to data $D$---typicality, simulation, prediction, decision
- From $D$ to $M$---estimation, inference
- Model similarity criterion
- End-to-end consistency!
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Foundations Matter!

- It is not about neatly organizing what we all agree on!
- Foundational choices have real-world consequences.
- Foundations of probability are much closer to the surface of the applications of probability than are the foundations of arithmetic to accounting or the foundations of physics to physics (Suppes 98) or to engineering.
Disagreements over foundations lie at the root of controversies over concepts of probability and statistical practice.

What are the boundaries of probabilistic reasoning?

How should we develop its content?
Theories of Probability (ToP)

- What counts as a theory of probability and what does not?
- Not everything that can be graded is probability.
- Single concept of probability cannot accommodate to all realms of discourse and to all purposes.
- Imperialist/Dogmatic claims---eliminating the feet that do not fit the shoe.
Desiderata for a ToP

- Probability is graded potential, propensity, belief, or evidential support.
- Gradations of potential or propensity exceed their evidential bases.
- But, this is also true of physical laws.
- Are there evidential bases for grading strength of beliefs?
Gradation monotonic with respect to implication or event containment.
But also …[cancellation, extremal elements]
How fine-grained is the gradation?
Is probability fundamental or merely supervenient or a placeholder?
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The Ruling Triumvirate

- Meaning, Measurement, Mathematics

Diagram:
- Meaning
- Precision/Gradation
- Axiomatics

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Meaning

- Primary importance -- use of word "interpretation" subordinates it to axioms
- Philosophers have a say here
- Subjective/individual (gradation of belief)
- Objective logical/formal (gradation of epistemic support)
- Objective empirical/scientific (gradation of tendency to occur)
- What else?
Attending to the Objective/Empirical

- ISIPTA and others have shown a marked preference for the subjective and personalistic view of probability.
- The subjective view profits from the mind’s limited insight into the brain.
- PROBABILITY DOES EXIST
- Objective empirical probability is far too important to suffer neglect.
Zur Quantenmechanik der Stoßvorgänge.

[Veröffentlichung: 1926]

Von Max Born.

November 1926.

Durch die Entwicklung der Quantenmechanik sind die Anfänge ergriffen, die die Quantenmechanik in der Schaffung neuer Formen nicht nur die mechanischen Grundlagen, sondern auch die Quantenmechanik zu beeinflussen.

Die von Heisenberg begründete Quantenmechanik hat bisher verschiedenen Bewegungen zur Annahme der quantenmechanischen Grundlagen und der mit ihnen verknüpften Veränderungen in der Natur der Welt, insbesondere der Quantenmechanik, die Veranlassung gegeben, die Quantenmechanik zu einer neuen Form der Quantenmechanik zu entwickeln.

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Max Born (June 1926)
Norbert Wiener
Wiesner, Lee, Wiener
Representation--
Precision/Gradation

- "At the margins of precision, the Universe flickers." (M. Black)
- How to think about measurement scales
- Empirical relational systems
- Mathematical relational systems
- Homomorphism for relations of likelihood, disjointness, and independence
- $P(A)$ is a real number and arithmetic operations $+, \times$ are meaningful
Precision/Gradation

- P is interval-valued (prices?)
- P is a set of measures (meaning?)
- P is fuzzy-valued
- Probability is a binary relation of (partial) ordering--P maps events into a (partially) ordered set--``A is at least as probable as B’’
- Probability is modal (``probably’’) or classificatory
Axioms and Mathematics

- Sine qua non or seduction?
- Good sense and rigor can be uneasy partners.
- Variety of realms and variety of meanings require a variety of axiom systems.
- Axioms are a tool for rendering overly precise our qualitative commitments to meaning and precision so that they can then be understood and explored.
- What don’t the axioms tell us? Many models
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``Take-Home” Thoughts

- The foundations of probability impact directly the applications of probability.
- Study of meaning (interpretation) is essential.
- Conscious choice of precision/gradation, matched to empirical phenomenon, is essential.
- Pride in imprecision, coarse-grainedness!
- Axioms and mathematics should not be an end in themselves in probability.
- Imprecise probability should not neglect objective physical/empirical chance.
The Future of Foundations