# The role of generalised p-boxes in imprecise probability theories

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# Introducing Didier

## Position

CNRS Research advisor, IRIT, Toulouse

## Some (scientific) interests

- Numerical possibility theory / imprecise probability
- Qualitative possibilistic logic
- Information fusion
- Fuzzy sets, rough sets, fuzzy interval analysis, fuzzy rules, . . .
- . . .

# Introducing me

## Position

2005 - 2008 - Phd student at the Institute of radiological protection and nuclear safety: uncertainty representation and treatment

2009 permanent researcher at French agricultural research centre working for international development

## Main interests

- uncertainty modeling
- uncertainty fusion
- uncertainty propagation and independence modelling
- (soon) learning methods and graphical models

# Current situation

# Current situation: applicative fields

→ agronomical chains, from crop to final product.

- wheat durum


→

→



- natural rubber (from Hevea tree)


→

→


# Current situation: long terms goals

Build a muticriteria decision model of the whole chain, basing the modelling on:

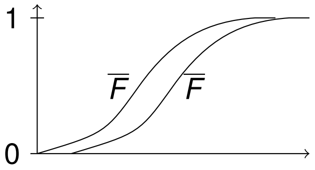
1. Expert knowledge (possibly modelled by IP)
2. Experimental data (through learning)
3. Physical models when available

# Motivation: two useful models

## P-boxes

Lower and upper prob. bounds over $(-\infty, x]$.

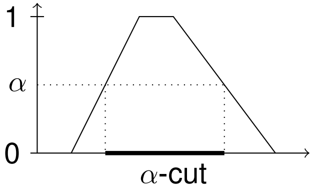
Useful for expert elicitation, risk analysis, reliability analysis, . . .




## Possibilities

Lower prob bounds given to nested sets

Useful for expert elicitation, linguistic assessments, . . .



# Generalised p-boxes: basic ideas

Take both attributes of possibility distributions and p-boxes to extend the to a single model, expressible by lower/upper probabilistic bounds over nested sets and represented by two distributions $\underline{F}, \overline{F}$

Set of constraints

$$\alpha_i \leq P(A_i) \leq \beta_i$$

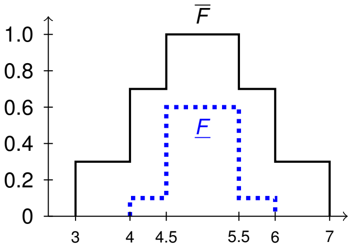
$$A_1 \subseteq A_2 \subseteq \ldots A_N \subseteq X$$

Example :

$0.3 \leq P(\mathrm{pH} \in [4.5, 5.5]) \leq 0.6;$

$0.7 \leq P(\mathrm{pH} \in [4, 6]) \leq 0.9;$

$1 \leq P(\mathrm{pH} \in [3, 7]) \leq 1.$



# Links with other models

- **Credal sets:** a gen. p-box induce a particular credal set $\mathcal{P} = \{P | \alpha_i \leq P(A_i) \leq \beta_i\}$
- **Random sets:** a gen. p-box induce an $\infty$-monotone lower probability, and can therefore be seen as a random set
- **Possibility distributions:** the credal set of any gen. p-box can be described by the intersection of credal sets induced by two possibility distributions

$\rightarrow$ to know to which extent this model is useful, need to consider its behavior and computational aspects for different treatments (conditioning, fusion, . . . )

# Treatments we've been looking at

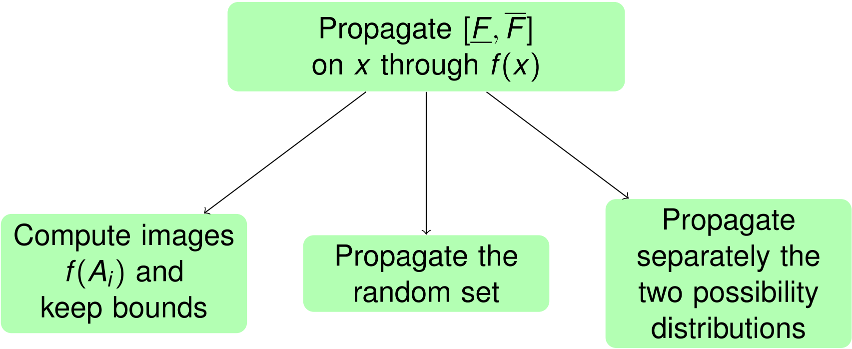
- Computing lower probabilities
- Propagation through (det.) function $f$
- Conditioning
- Merging

# Computation of probabilistic bounds

**For a given event $A$, how to compute $\underline{P}(A)$**

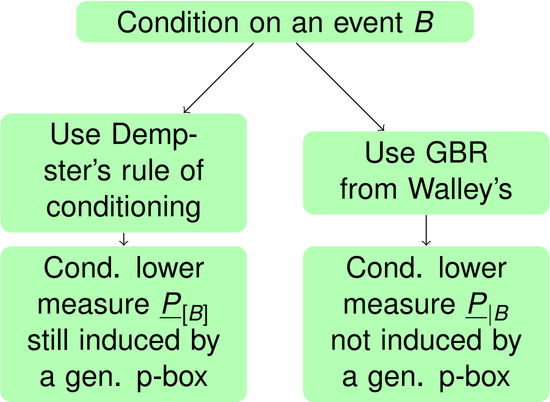
Easy due to the additivity of lower probability $\underline{P}$ on specific collections of sets

# Propagation




$\longrightarrow$ result usually not a gen. p-box

# Conditioning



# Merging (idempotent rules)

Merging $S$ different gen. p-boxes $[\underline{F}, \overline{F}]_1, \ldots, [\underline{F}, \overline{F}]_S$

**Conjunctive**

$$\underline{F}_{\cap}(x) = \max_{i=1,S} \underline{F}_i(x)$$
$$\overline{F}_{\cap}(x) = \min_{i=1,S} \overline{F}_i(x).$$

**Disjunctive**

$$\underline{F}_{\cup}(x) = \min_{i=1,S} \underline{F}_i(x)$$
$$\overline{F}_{\cup}(x) = \max_{i=1,S} \overline{F}_i(x)$$

**Arith. mean**

$$\underline{F}_{\Sigma}(x) = \sum_{i=1}^{S} \lambda_i \underline{F}_i(x)$$
$$\overline{F}_{\Sigma}(x) = \sum_{i=1}^{S} \lambda_i \overline{F}_i(x)$$

$\rightarrow$ Coherent with merging of usual p-boxes and possibility distributions
$\rightarrow$ Except under specific conditions, the result is not a gen. p-box, but a cloud (Neumaier, 04)

# Conclusions

Main interests of gen. p-boxes lie in

- the elicitation and representation of information
- the use of it as (relatively gross) approximations
- the use of convex confidence regions to simplify problems (Fuchs, Neumaier, 09)

**More details are on the poster!**