Object association in the TBM framework, application to vehicle driving aid

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Univ. Lille nord de France, UArtois, LGI2A

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Outline

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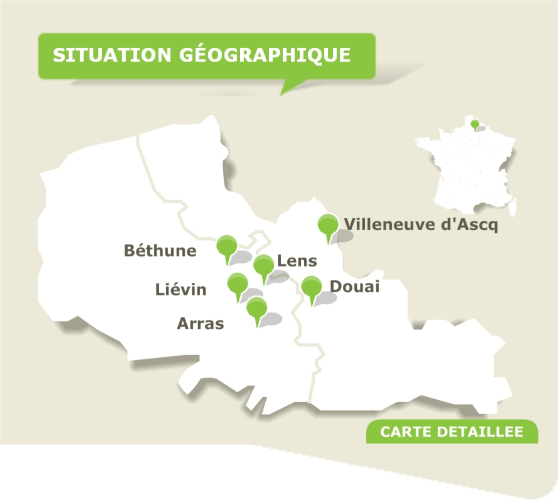
University of Artois

A multi-polar university in northern France



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The authors

- David Mercier

  ISIPTA summer school of Lugano (2004),
  PhD University of Compiègne (2006),
  Associate professor, University of Artois (2008)
  Information Fusion, Transferable Belief Model.

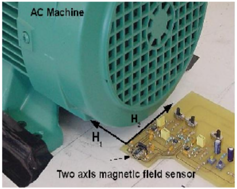
- Éric Lefèvre

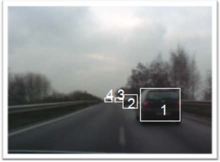
  PhD University of Rouen (2001),
  Associate professor, University of Artois (2002),
  Information Fusion, Transferable Belief Model.

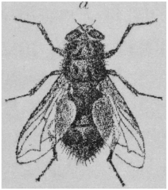
- Daniel Jolly

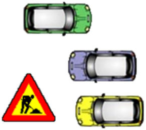
  PhD University of Lille (1976),
  Professor, University of Artois (1999),
  LGI2A Director since 2003,
  Information Fusion, Complex System Modeling, Road Traffic.

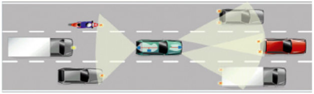
Examples of fusion applications developed within the LGI2A











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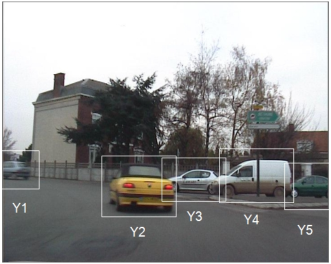

Vehicles detected at time step t (known objects)

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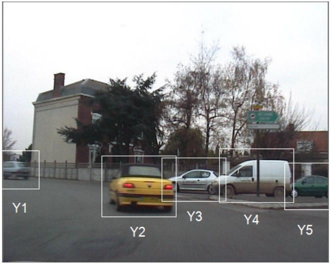

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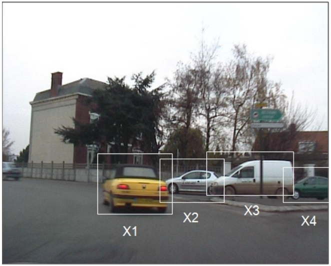

Vehicles detected at time step $t+1$ (perceived objects)

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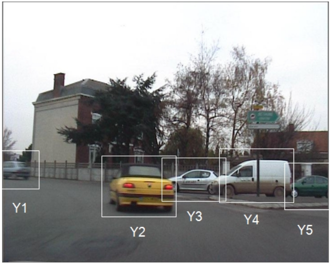

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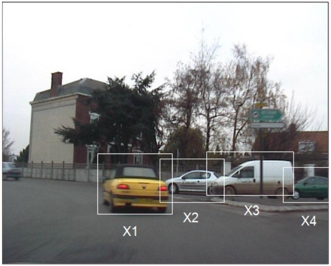

Vehicles detected at time step $t + 1$ (perceived objects)

Question to answer:

- What are the relations between known objects and perceived objects?

Available information and modeling

- Uncertain and imprecise information regarding the association of each perceived object X_i and with each known object Y_j.
- Example:
  - Object X_1 corresponds to object Y_2 with some degree of belief...
- Modeling with belief functions.

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Available information and modeling

- Uncertain and imprecise information regarding the association of each perceived object X_i and with each known object Y_j.
- Example:
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- Modeling with belief functions.
- Belief functions used in different theories of uncertainty. Different semantics:
  - Models based on lower and upper probabilities including Dempster's model and the Hints model;
  - Random set theory;
  - Transferable Belief Model (TBM): belief functions interpretated as weighted opinions.
- Last model adopted in the paper.

Outline

1. Research unit (LGI2A)
2. Association problem
3. Transferable Belief Model (TBM): basic notions and notation
4. A modeling in the TBM framework

Transferable Belief Model

- A model of uncertain reasoning and decision-making based on two levels:
  - **Credal level**: available pieces of information are represented by belief functions, and manipulated;
  - **Decision level**: belief functions are transformed into probability measures when a decision has to be made, and the expected utility is maximized.

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Representing the information
Mass Function (MF)

- Frame of discernment:

  $\Omega = \{\omega_1, \ldots, \omega_K\}$: a finite set of possible answers to a given question Q.

- Mass function:

  Information held by an agent Ag regarding the answer to question Q can be quantified by a mass function m^{Ω}_{Ag}, defined as a function from 2^{Ω} to $[0, 1]$, and verifying:

$$\sum_{A \subseteq \Omega} m^{\Omega}_{Ag}(A) = 1 \ .$$

  Subset A s.t. $m(A) > 0$ is called a *focal set* of m.

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Manipulating the information

Vacuous extension and conjunctive rule of combination (CRC)

- Vacuous extension:

  Convey a mass function m^{Θ} to a finer frame Ω (a refinement):

$$m^{\Theta\uparrow\Omega}(\rho(A)) = m^{\Theta}(A), \quad \forall A \subseteq \Theta ,$$

  where ρ is the refining of Θ in Ω.

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  where ρ is the refining of Θ in Ω.

- Conjunctive rule of combination (CRC):

  Two distinct and reliable MFs m_1 and m_2 can be combined using the CRC:

  $$m_1 \textcircled{\cap} m_2(A) = \sum_{B\cap C=A} m_1(B)m_2(C), \quad \forall A \subseteq \Omega.$$

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Decision making

- Choose the decision d among a set of possible decisions $\mathcal{D}$, which minimizes the *expected risk* defined by:

$$R(d) = \sum_{\omega \in \Omega} c(d, \omega) BetP^{\Omega}(\{\omega\}),$$

with

- $c : \mathcal{D} \times \Omega \rightarrow \mathbb{R}$ a cost function.
- $BetP^{\Omega}$: pignistic transformation of m^{Ω}:

$$BetP^{\Omega}(\{\omega\}) = \sum_{\{A \subseteq \Omega, \omega \in A\}} \frac{m(A)}{|A|\,(1 - m(\emptyset))}, \quad \forall \omega \in \Omega.$$

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Modeling

- Frames of discernment involved:
  - $\Omega_{i,j} = \{y_{i,j}, n_{i,j}\}$: two possible answers (yes or no) to the question "Is the perceived object X_i associated with the known object Y_j?";
  - $\Omega_{X_i} = \{Y_1, Y_2, \ldots, Y_M, ^\star\} = \{1, 2, \ldots, M, ^\star\}$: answers to the question "Which object is associated with X_i?", proposition "$\star$" meaning an unknown object.

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- Algorithm:
  - Express each piece of information $m^{\Omega_{i,j}}$ on the common frame Ω_{X_i}: $m^{\Omega_{i,j}\uparrow\Omega_{X_i}}$, denoted $m_j^{\Omega_{X_i}}$;
  - Combine conjunctively these MFs. Let us denote $m^{\Omega_{X_i}}$ this result;
  - Chosen decision = the association maximizing the probability $BetP^{\Omega_{X_1}\times\ldots\times\Omega_{X_N}}$ (and verifying some constraints of association).

Example 1/2

1 perceived object X_1 and 2 known objects Y_1 and Y_2 s.t.:

$$\begin{cases} m^{\Omega_{1,1}}(\{y_{1,1}\}) &= .2 \\ m^{\Omega_{1,1}}(\{n_{1,1}\}) &= .45 \\ m^{\Omega_{1,1}}(\Omega_{1,1}) &= .35 \end{cases} \qquad \begin{cases} m^{\Omega_{1,2}}(\{y_{1,2}\}) &= .45 \\ m^{\Omega_{1,2}}(\{n_{1,2}\}) &= .15 \\ m^{\Omega_{1,2}}(\Omega_{1,2}) &= .4 \end{cases}$$

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Expressing this information on Ω_{X_1}:

$$\begin{cases} m_1^{\Omega_{X_1}}(\{1\}) &= .2 \\ m_1^{\Omega_{X_1}}(\overline{\{1\}}) &= .45 \\ m_1^{\Omega_{X_1}}(\Omega_{X_1}) &= .35 \end{cases} \qquad \begin{cases} m_2^{\Omega_{X_1}}(\{2\}) &= .45 \\ m_2^{\Omega_{X_1}}(\overline{\{2\}}) &= .15 \\ m_2^{\Omega_{X_1}}(\Omega_{X_1}) &= .4 \end{cases}$$

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After the CRC:

A	$\emptyset$	$\{1\}$	$\{2\}$	$\{\star\}$	$\{1,\star\}$	$\{2,\star\}$	$\{1,2,\star\}$
$m^{\Omega_{X_1}}(A)$	.09	.11	.36	.07	.05	.18	.14
$BetP^{\Omega_{X_1}}(A)$		.20	.55	.25			

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Conclusion: X_1 **is associated with** Y_2, Y_1 **has disappeared.**

Example 2/2

On the other hand: possible to express the information on Ω_{Y_1} and Ω_{Y_2}:

$$\begin{cases} m_1^{\Omega_{Y_1}}(\{1\}) &= .2 \\ m_1^{\Omega_{Y_1}}(\overline{\{1\}}) &= .45 \\ m_1^{\Omega_{Y_1}}(\Omega_{Y_1}) &= .35 \end{cases} \qquad \begin{cases} m_1^{\Omega_{Y_2}}(\{1\}) &= .45 \\ m_1^{\Omega_{Y_2}}(\overline{\{1\}}) &= .15 \\ m_1^{\Omega_{Y_2}}(\Omega_{Y_2}) &= .4 \end{cases}$$

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A	$\emptyset$	$\{1\}$	$\{\star\}$	$\{1,\star\}$
$m^{\Omega_{Y_1}}(A)$		.2	.45	.35
$BetP^{\Omega_{Y_1}}(A)$		.375	.625	1
$m^{\Omega_{Y_2}}(A)$		.45	.15	.4
$BetP^{\Omega_{Y_2}}(A)$		.65	.35	1

- $BetP^{\Omega_{Y_1}\times\Omega_{Y_2}}(\{1,\star\}) = .375 \times .35 = .131$;
- $BetP^{\Omega_{Y_1}\times\Omega_{Y_2}}(\{\star,1\}) = .625 \times .65 = .406$;
- $BetP^{\Omega_{Y_1}\times\Omega_{Y_2}}(\{\star,\star\}) = .625 \times .35 = .219$,

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Same results, unfortunately not always the case...

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Thank you for your attention.