On the Behavior of the Robust Bayesian Combination Operator and the Significance of Discounting

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Outline

- Research context: information fusion
- The robust Bayesian combination operator
  - Imprecision
  - Conflict
  - Discounting
  - Examples
- Contributions

Research Context: Information Fusion

- Information fusion research program, University of Skövde
- Information fusion
  - Reasoning under uncertainty, signal processing, logic, ontologies, etc.
  - Application oriented
    - Defense, Robotics, and Computer security, etc.
  - The combination problem
    - Evidence theory
    - Credal sets

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Combination and Aggregation Operators

- Combination operator
  - Operands are “different” pieces of evidences for some state space
  - Identical operands typically reinforces the resulting evidence for some state
- Aggregation operator
  - Operands can be evidences, belief, etc.
  - Result represent a common agreement of the operands
  - Identical operands give the operand as result (perfect agreement)

(Halpern and Fagin 1992), (Torra and Narukawa 2007)

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The Bayesian Combination Operator

From Bayes theorem, we can derive the Bayesian combination (BC) operator:

$$p_1(X) \otimes_{\mathcal{B}} p_2(X) \triangleq \frac{p_1(X)p_2(X)}{\sum_{x \in \Omega_X} p_1(x)p_2(x)}$$

where $p_1(X)$ and $p_2(X)$ are probability functions (normalized likelihoods).

(Arnborg 2004), (Arnborg 2006)

The Robust Bayesian Combination (RBC) Operator

The Robust Bayesian Combination (RBC) Operator:

$$\mathcal{P}_X^1 \otimes_{\mathcal{R}} \mathcal{P}_X^2 \triangleq CH\left\{ p_i(X) \otimes_{\mathcal{B}} p_j(X) : \; p_i \in \mathcal{P}_X^1, \, p_j \in \mathcal{P}_X^2 \right\}$$

where CH denotes the convex hull and $\mathcal{P}_X^1$, $\mathcal{P}_X^2$ are credal sets (convex sets of normalized likelihoods).

(Arnborg 2004), (Arnborg 2006)

Theorem: $\mathcal{P}_X^1 \otimes_{\mathcal{R}} \mathcal{P}_X^2 = ext(\mathcal{P}_X^1) \otimes_{\mathcal{R}} ext(\mathcal{P}_X^2)$

Proof: See paper.

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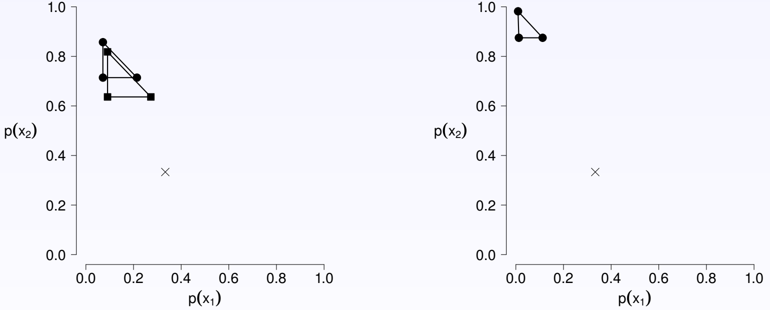
Example: The RBC Operator



(a) $\mathcal{P}_X^1$ (circles) and $\mathcal{P}_X^2$ (squares)　　(b) $\mathcal{P}_X^{1,2}$

Figure: $\mathcal{P}_X^i$, $i \in \{1,2\}$, and $\mathcal{P}_X^{1,2}$, $\Omega_X = \{x_1, x_2, x_3\}$

Degree of Imprecision:

$$\mathcal{I}(\mathcal{P}_X) \triangleq \frac{1}{n} \sum_{x \in \Omega_X} \Delta(x)$$

where $\Delta(x)$ is Walley's measure of degree of imprecision for a single event $x \in \Omega_X$:

$$\Delta(x) \triangleq \max_{p \in \mathcal{P}_X} p(x) - \min_{p \in \mathcal{P}_X} p(x)$$

(Walley 1991)

Degree of Conflict:

$$\mathcal{K}(\mathcal{P}_X^1, \mathcal{P}_X^2) \triangleq \frac{\mathcal{H}(\mathcal{P}_X^1, \mathcal{P}_X^2)}{\sqrt{2}},$$

where $\mathcal{H}$ is the Hausdorff distance defined by:

$$\mathcal{H}(\mathcal{P}_X^1, \mathcal{P}_X^2) \triangleq \max\left\{\overrightarrow{\mathcal{H}}(\mathcal{P}_X^1, \mathcal{P}_X^2), \overrightarrow{\mathcal{H}}(\mathcal{P}_X^2, \mathcal{P}_X^1)\right\},$$

and $\overrightarrow{\mathcal{H}}$ is the forward Hausdorff distance defined as:

$$\overrightarrow{\mathcal{H}}(\mathcal{F}_1, \mathcal{F}_2) \triangleq \max_{f_i \in \mathcal{F}_1}\left\{\min_{f_j \in \mathcal{F}_2} d(f_i, f_j)\right\},$$

where d is the Euclidean distance.

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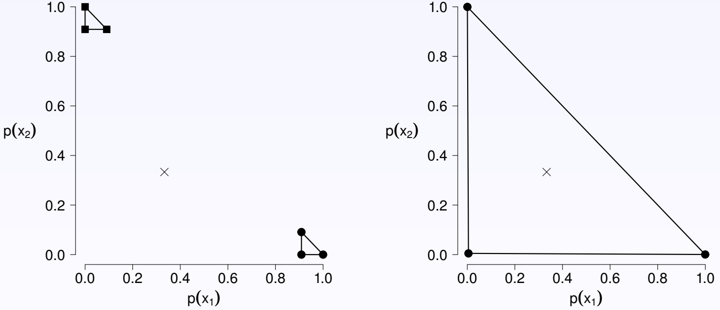


(a) $\mathcal{P}_X^1$ (circles) and $\mathcal{P}_X^2$ (squares) (b) $\mathcal{P}_X^{1,2}$

Figure: $\mathcal{P}_X^i$, $i \in \{1, 2\}$, and $\mathcal{P}_X^{1,2}$, $\Omega_X = \{x_1, x_2, x_3\}$

The RBC Discounting Operator:

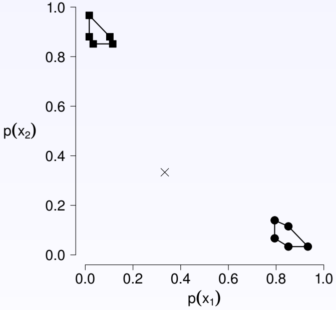
$$\mathcal{D}(\mathcal{P}_X, \mathcal{W}) \triangleq CH\left\{wp + (1-w)p_u : w \in \mathcal{W}, p \in \mathcal{P}_X\right\},$$

where $\mathcal{P}_X \subseteq \mathbb{R}^n$, $\mathcal{W} \subseteq [0,1]$ is an interval of reliability weights, and $p_u \in \mathbb{R}^n$, $n = |\Omega_X|$, is the uniform distribution over Ω_X.

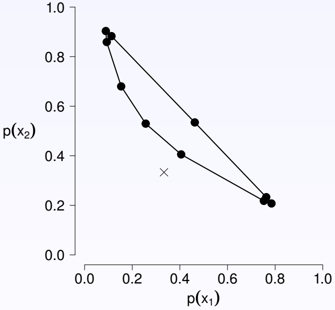
Theorem: $\mathcal{D}(\mathcal{P}_X, \mathcal{W}) = \mathcal{D}(ext\left(\mathcal{P}_X\right), ext\left(\mathcal{W}\right))$

Proof: See paper.

Example: Discounting



(a) $\mathcal{D}\left(\mathcal{P}_X^1, \mathcal{W}_1\right)$ (circles) and $\mathcal{D}\left(\mathcal{P}_X^2, \mathcal{W}_2\right)$ (squares)



(b) $\mathcal{P}_X^{1_d,2_d}$

Figure: $\mathcal{W}_1 = [0.80, 0.90]$, $\mathcal{W}_2 = [0.90, 0.95]$

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Contributions

- Extreme-point proof for the RBC operator
- Measures
  - Degree of imprecision (Walley)
  - Degree of conflict (Hausdorff)
- The results of the RBC operator can be highly imprecise
- The RBC discounting operator
  - Interval of reliability weights
  - Computation on extreme points

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