

# **Bayes Linear Analysis of Imprecision in Computer Models, with Application to Understanding Galaxy Formation**

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\*work done in collaboration with Michael Goldstein (Dept. Mathematical Sciences); Richard Bower and Carlos Frenk's group at the Institute for Computational Cosmology, Durham University. Funding: MUCM and EPSRC.

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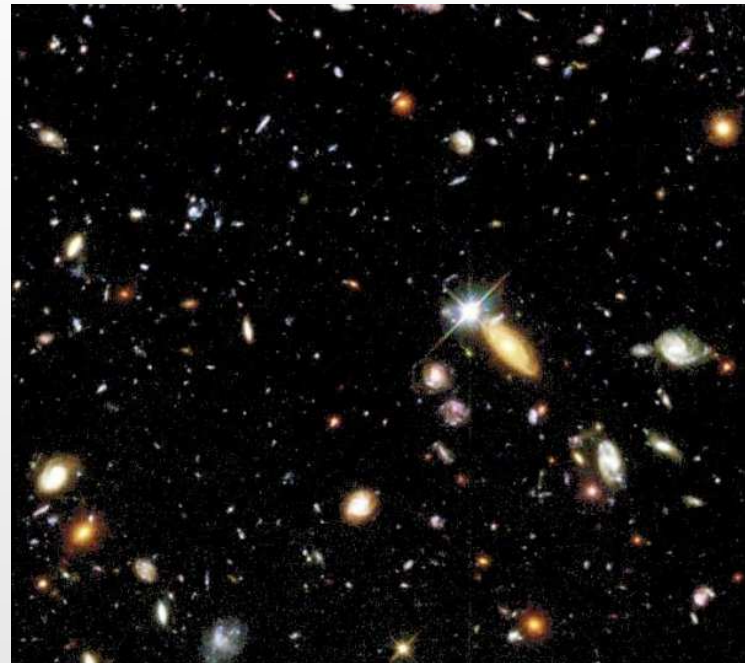
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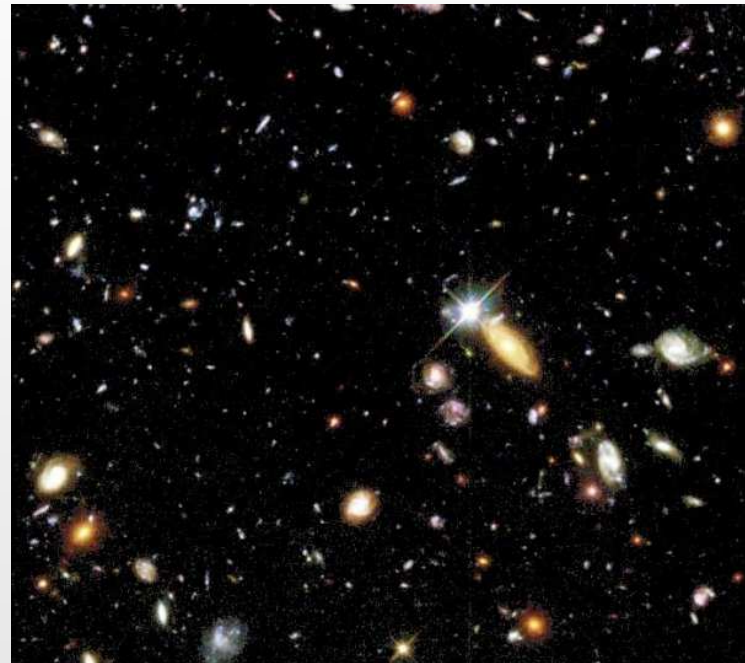
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- We treat the Model Discrepancy as Imprecise.
- Main goal is to understand the impact of such an Imprecise assessment on our ability to learn about the Input Uncertainty.

## Andromeda Galaxy and Hubble Deep Field View



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- Cosmologist want to understand the creation and evolution of Galaxies in the presence of large amounts of Dark Matter.

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- $\chi^*$  will be sensitive to the assessment of the Imprecise Model Discrepancy.
- Need to analyse in detail the impact of various features of the Imprecise Model Discrepancy on this set of acceptable inputs  $\chi^*$ .

## Input Parameters

- Due to expert judgements we attempt to History Match Galform over 8 of the input parameters (while taking into account the possible effects of the remaining 9).
- The input parameters and their initial ranges are:

**vhotdisk:** 100 - 550

**aReheat:** 0.2 - 1.2

**alphacool:** 0.2 - 1.2

**vhotburst:** 100 - 550

**epsilonStar:** 0.001 - 0.1

**stabledisk:** 0.65 - 0.95

**alphahot:** 2 - 3.7

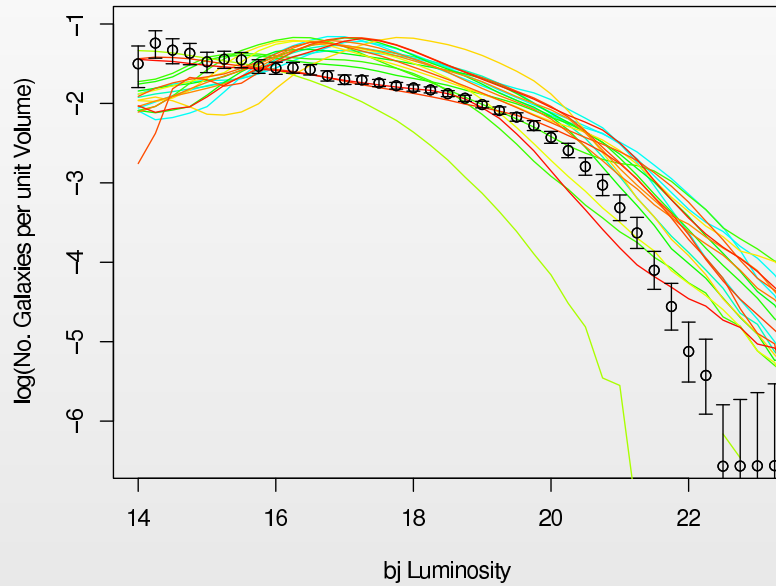
**yield:** 0.02 - 0.05

**What values should I choose  
to get 'good' outputs?**

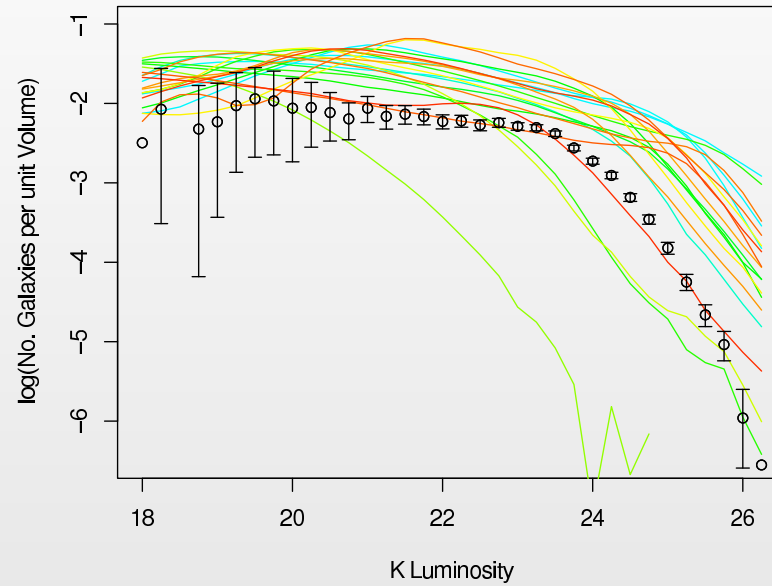
- The other 9 parameters are: VCUT, ZCUT, alphastar, tau0mrg, fellip, fburst, FSMBH, epsilonSMBHEddington and tdisk.

# Galform Outputs: The Luminosity Functions

bj Luminosity Function

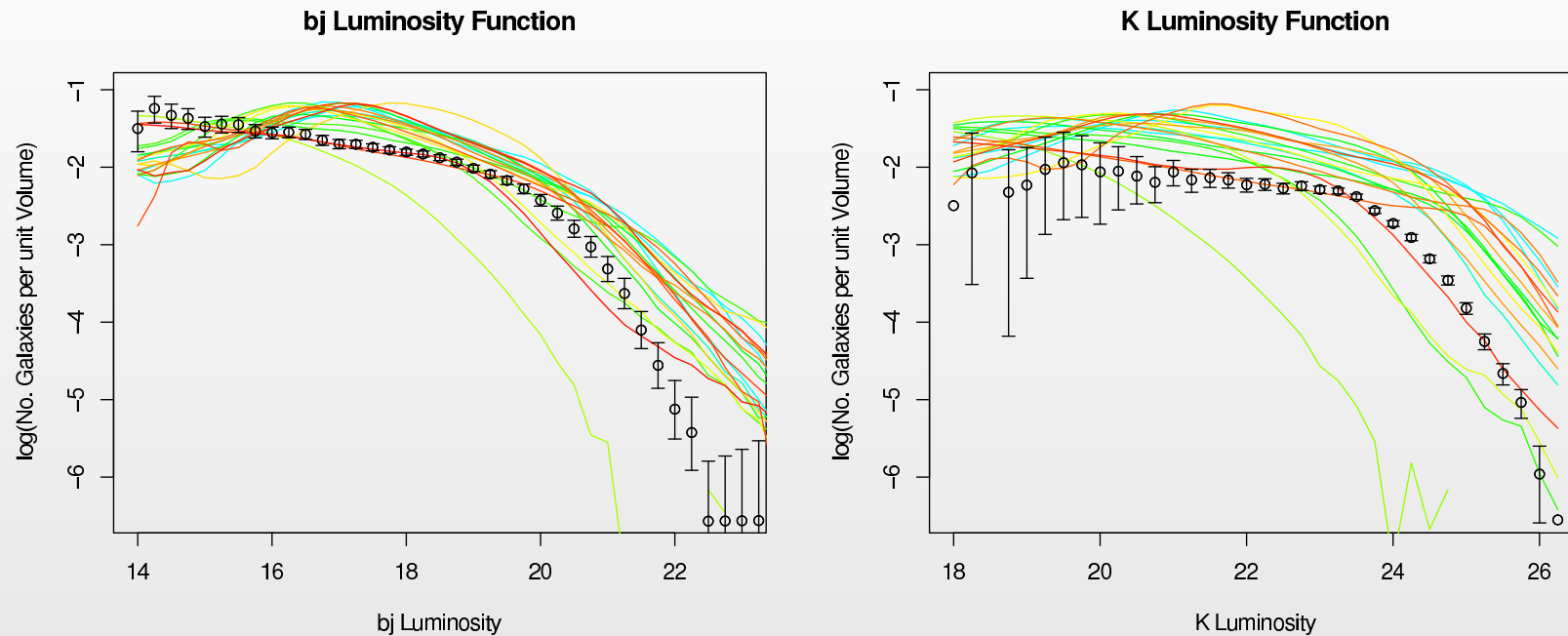


K Luminosity Function



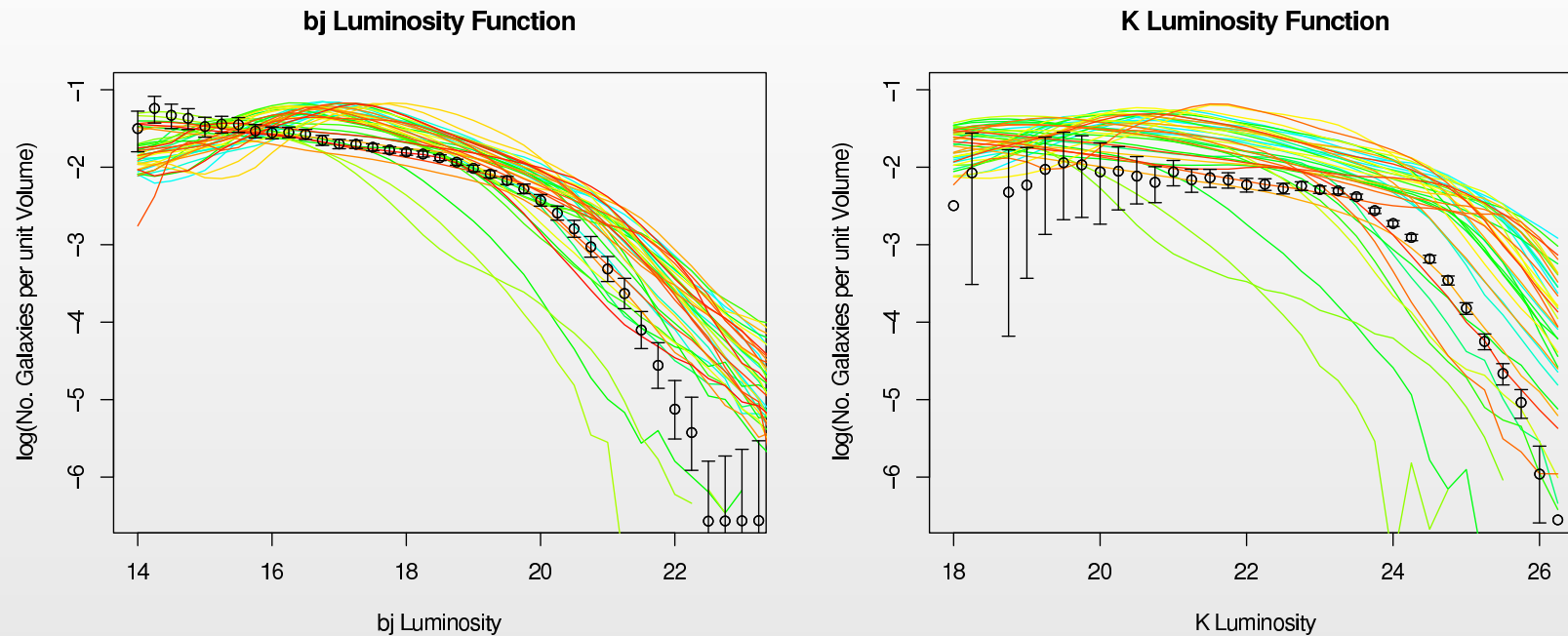
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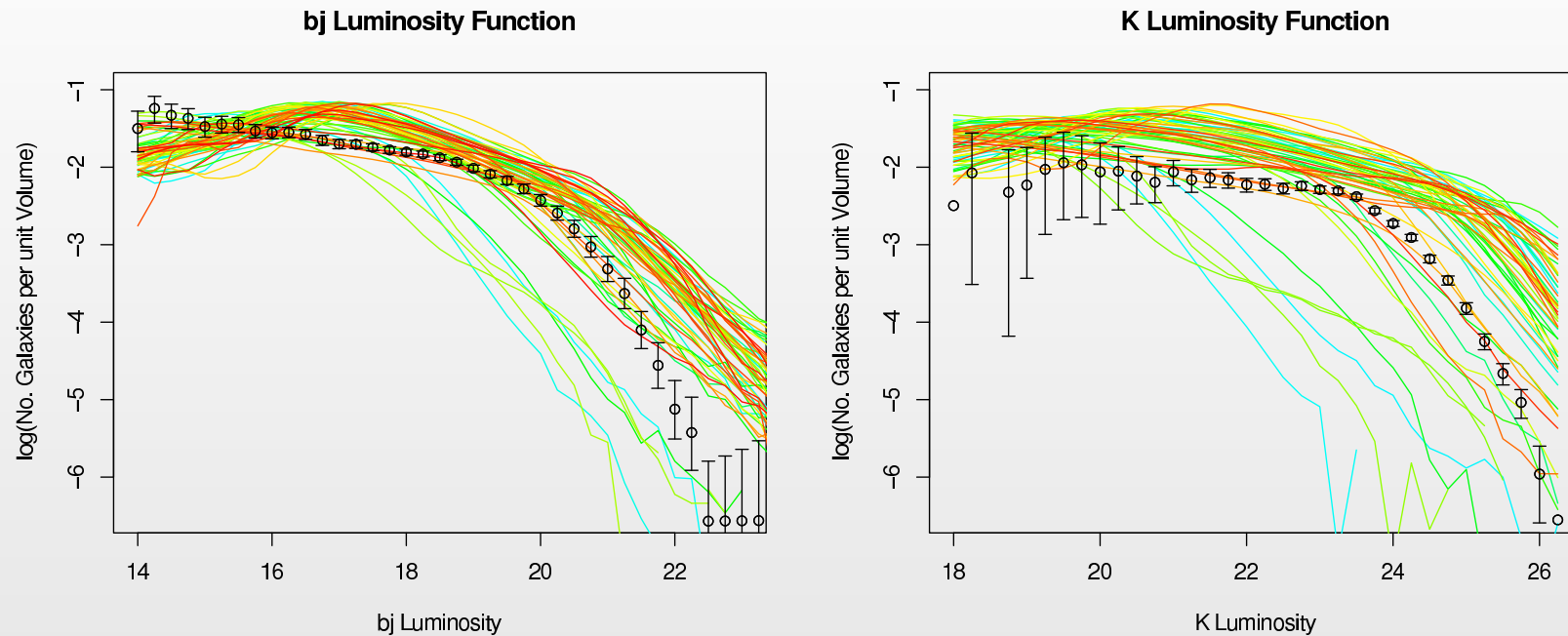
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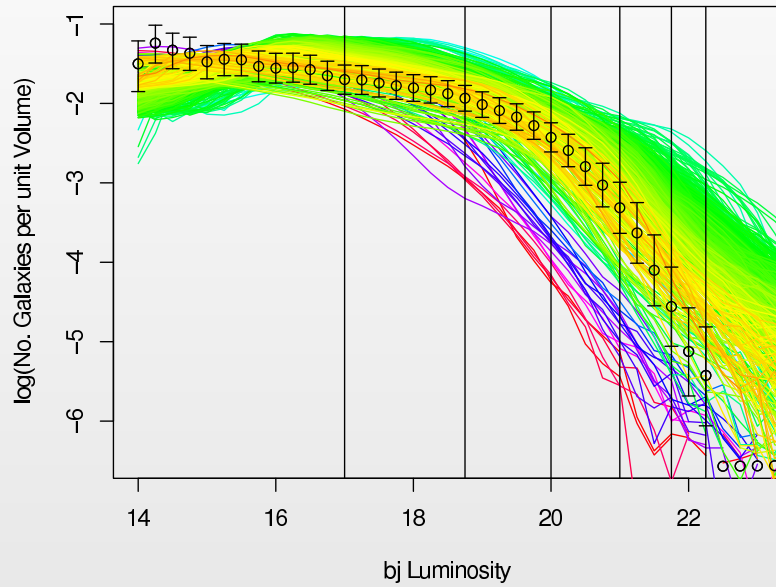
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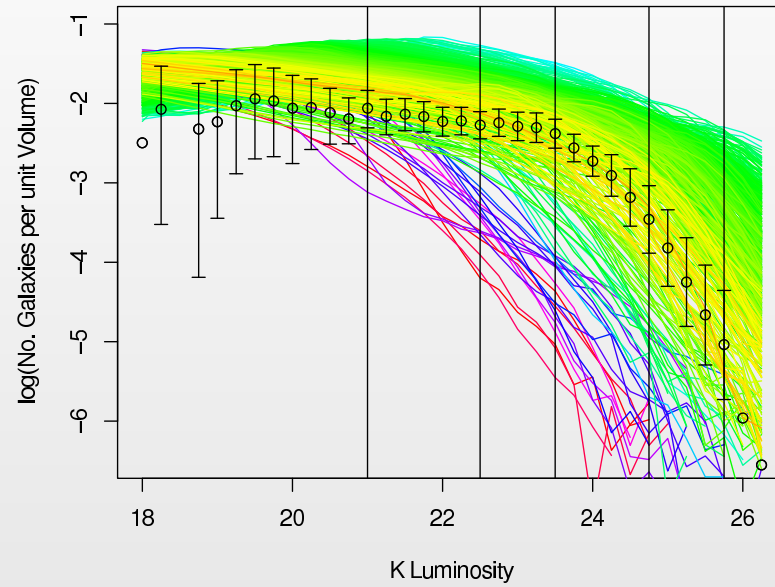
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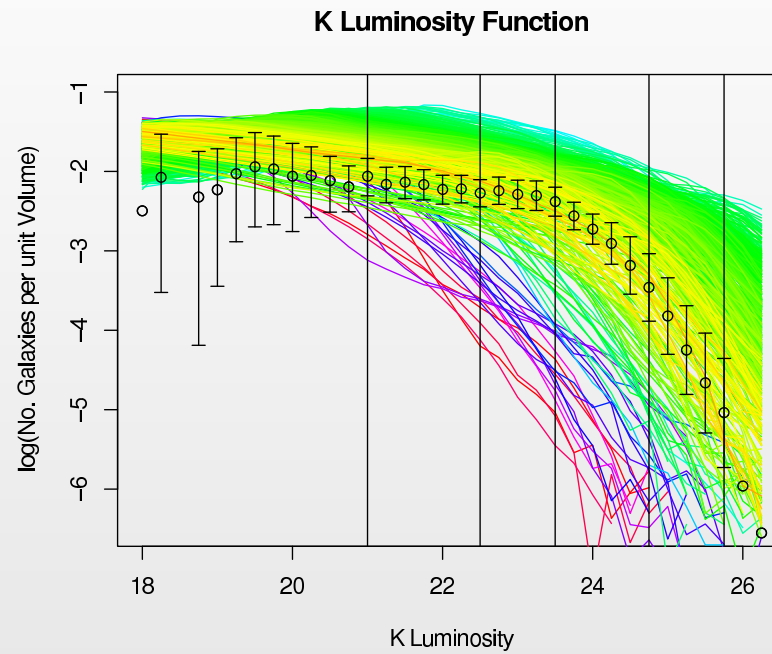
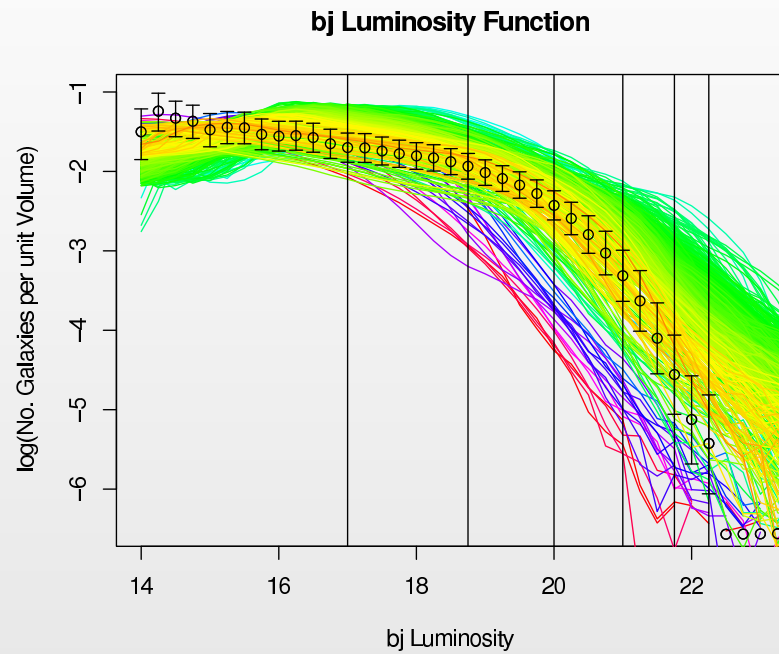


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- Outputs chosen to be informative enough to allow us to cut down the parameter space, but simple enough to be emulated easily.

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- We then link the real system denoted by  $y$  to the simulator by the equation:

$$y = f^* + \epsilon_{md},$$

where we define  $\epsilon_{md}$  to be the *model discrepancy* and assume that  $\epsilon_{md}$  is independent of  $f, x^*$ . (Here, and onwards, all probabilistic statements relate to the uncertainty judgements of the analyst.)

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- Finally, we relate the true system  $y$  to the observational data  $z$  by,

$$z = y + \epsilon_{obs},$$

where  $\epsilon_{obs}$  represent the observational errors.

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- Due to the complexity of a Computer Model analysis we employ Bayes Linear Methods, where (following de Finetti) we treat Expectation as primitive and only require specification of Expectations, Variances and Covariances.
- This makes assessment of the Model Discrepancy far easier than in a fully probabilistic analysis.

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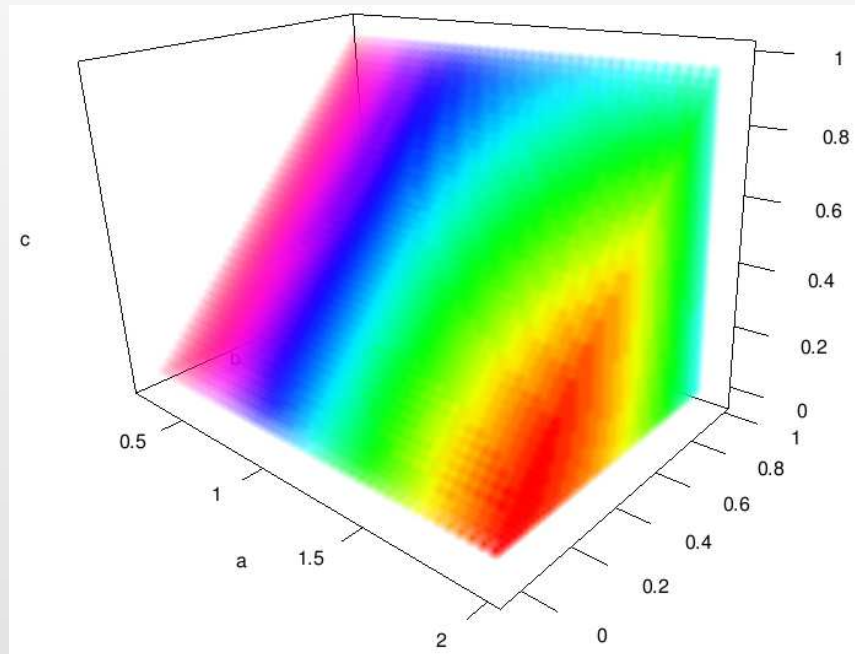
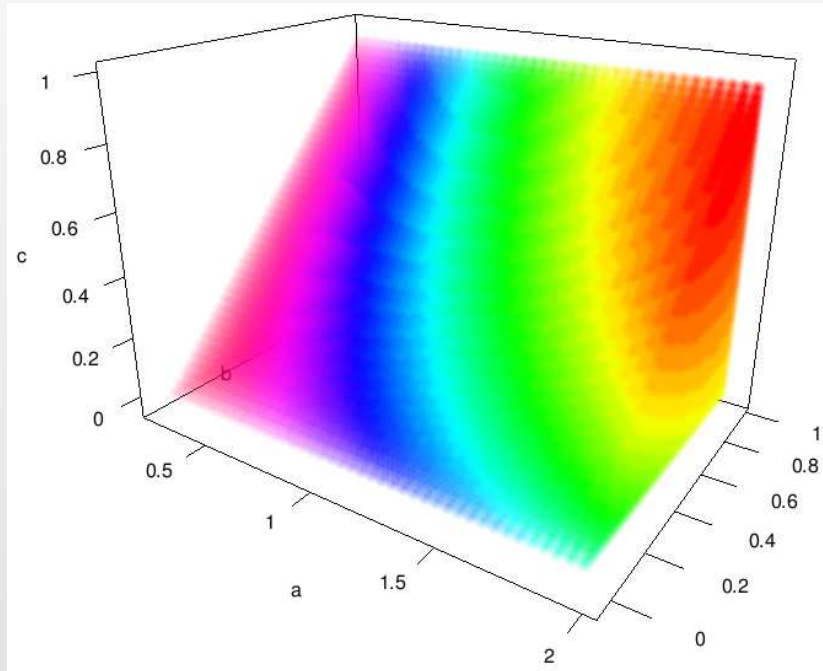
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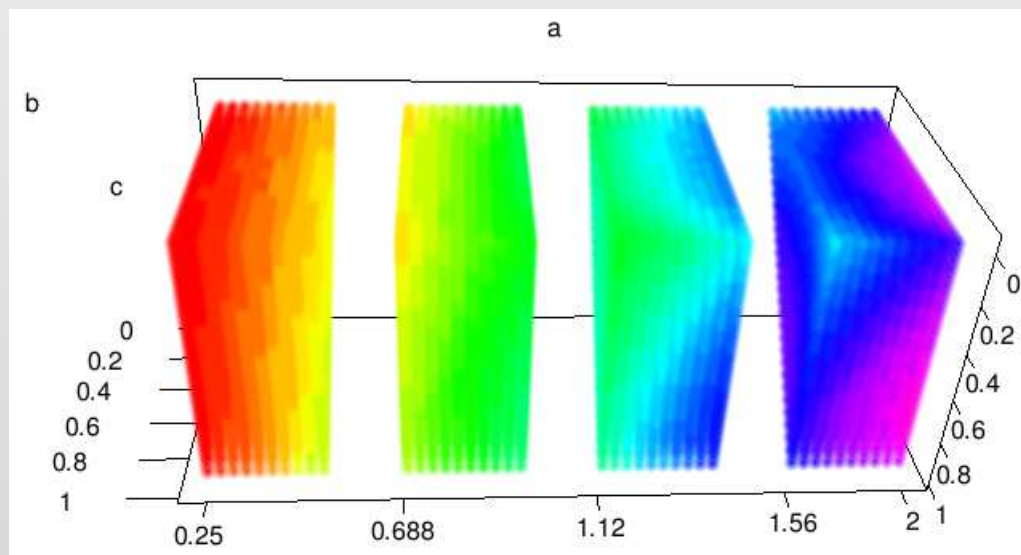
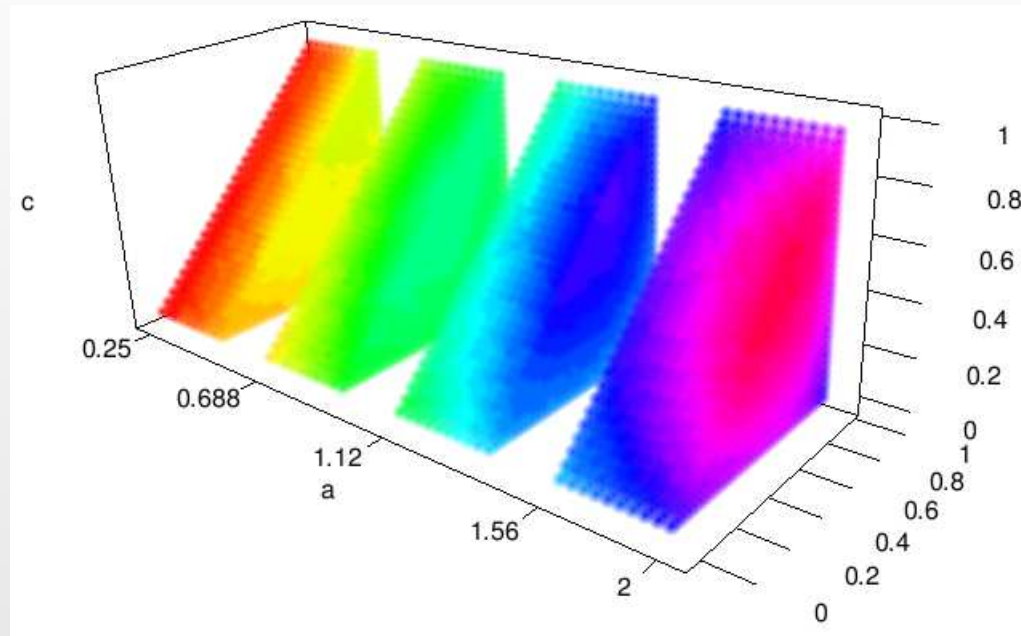
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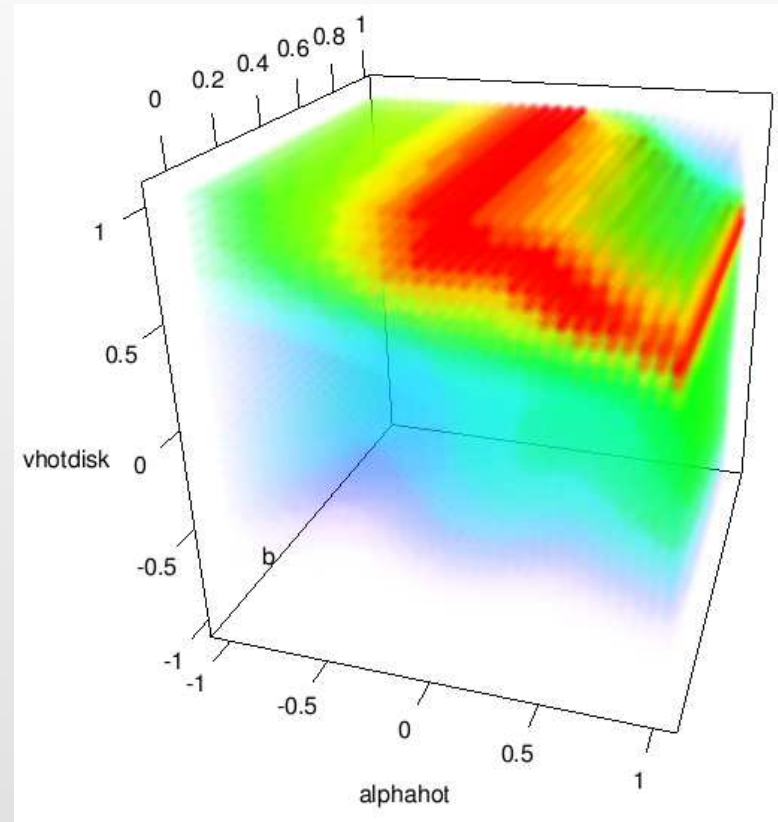
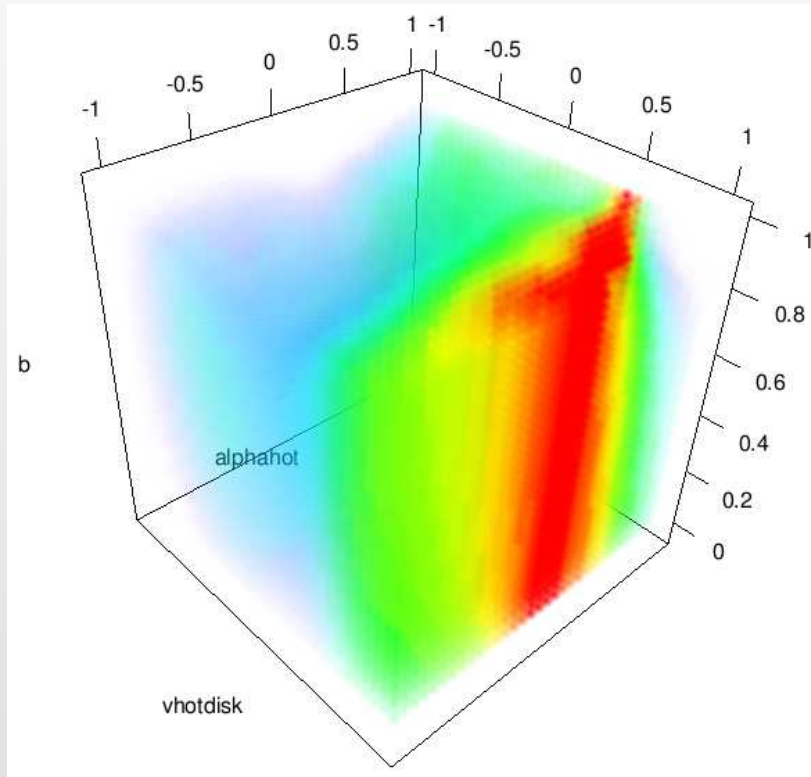
$I(x, a, b, c)$  with Fixed Inputs  $x$ .



# Percentage input space cut out.



# Required $a$ to Avoid Cutoff.



●

## References

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