Nonparametric Predictive System Reliability with Redundancy Allocation

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Abstract

Coolen [1] presented lower and upper probabilities for prediction of Bernoulli random quantities, which have strong internal consistency properties within the theory of interval probability. These lower and upper probabilities followed from an assumed underlying latent variable model, with future outcomes of random quantities related to data by Hill's assumption $A_{(n)}$ [3], and they are part of a wider statistical methodology called 'Nonparametric Predictive Inference' (NPI).

In a recent paper, Coolen-Schrijner, Coolen and MacPhee [2] presented nonparametric predictive inference for system reliability with redundancy allocation. They considered series systems, with each subsystem i a k_i-out-of-m_i system. The minimum number of components which are required to function in order for a subsystem i to function, k_i, is assumed to be fixed, while the total number of components in subsystem i, m_i, can be chosen to provide varying levels of redundancy. They applied NPI for Bernoulli data [1] to such systems, with inferences on each subsystem i based on information from tests on n_i components. The components tested are assumed to be exchangeable with the corresponding components to be used in that subsystem. Only situations where components, and therefore also the system, either function or not when called upon are considered.

Throughout this work, the lower probability of system functioning is used as a reliability measure. In [2], an attractive property for the subsystem reliability measure was proved, namely its log-concavity as function of the number of redundant components. This property allows rapid determination of an optimal sequence for allocation of redundant units over different subsystems. The existence of such a sequence implies that this model, if considered as a sequential decision problem, has a myopic optimal allocation policy. However, they only proved this result for tests in which no components failed.

We presented a generalization of this result to redundancy allocation following tests in which any number of components can have failed, which is an important generalization as redundancy is even more important when testing revealed some failing components. This presentation is based on results in [4].

Keywords. k-out-of-m systems; lower and upper probabilities; nonparametric predictive inference; redundancy allocation; series-parallel systems; system reliability.

References

[1] Coolen, F.P.A. (1998). Low structure imprecise predictive inference for Bayes' problem. *Statistics & Probability Letters* **36**, 349-357.

[2] Coolen-Schrijner, P., Coolen, F.P.A. and MacPhee, I.M. (2008). Nonparametric predictive inference for system reliability with redundancy allocation. *Journal of Risk and Reliability* **222**, 463-476.

[3] Hill, B.M. (1968). Posterior distribution of percentiles: Bayes' theorem for sampling from a population. *Journal of the American Statistical Association* **63**, 677-691.

[4] MacPhee, I.M., Coolen, F.P.A. and Aboalkhair, A. M. (2009). Nonparametric predictive system reliability with redundancy allocation following component testing. *Journal of Risk and Reliability*, to appear .