Flood defence infrastructure investment decisions using an imprecise probability model of mean sea level rise

Jim W Hall
Tyndall Centre for Climate Change Research
School of Civil Engineering and Geosciences
Newcastle University, UK
jim.hall@ncl.ac.uk

Hamish Harvey
School of Civil Engineering and Geosciences
Newcastle University, UK
hamish@hamishharvey.com

Abstract

Flood defence planning decisions are made on the basis of comparison of (whole life) cost of alternative investment options with the beneficial risk reduction that they are expected to yield. Consider a set of k options $\mathcal{Q} = \{q_i : i = 0, \dots, k-1\}$, each with an annual cost stream $C(i,0), \dots, C(i,t-1)$ over the t-year appraisal period. q_0 denotes a 'base case' in which $C(0,y) = 0 \; \forall y \in \{0, \dots, t-1\}$. The vector of n system state variables that determine the severity of flooding for any given option at any instant in time is denoted $\mathbf{x} \in \mathcal{X}$. For example, one of the most important variables influencing flood risk at a coastal site is sea level s (measured relative to land level). Short term variation in s is attributable to astronomic tides and meteorological elevation of the water level (surge). A given system state $\mathbf{x}$ will yield flood damage $D(\mathbf{x}) \geq 0$, which is measured in the same economic units as C. We take variation in $\mathbf{x}$ in any given year y to be stationary in time, and described by a jpdf $f_{i,y}(\mathbf{x})$. The flood risk $R(i,y)$ associated with option q_i in year y is the expected annual damage: $R(i,y) = \int_{\mathcal{X}} D(\mathbf{x}) f_{i,y}(\mathbf{x}) d\mathbf{x}$. The benefit of option q_i in year y is taken as the risk reduction relative to the base case i.e. $R(0,y) - R(i,y)$, so that the expected utility for any given year is $E(i,y) = R(0,y) - R(i,y) - C(i,y)$. This utility is discounted over the appraisal period to yield a Net Present Value $N(i)$ for option q_i: $N(i) = \sum_{y=0}^{t-1} \frac{1}{(1+e)^y} E(i,y)$, where e is the discount rate. A risk-neutral decision maker who wishes to maximise Net Present Value of expected utility chooses the option $q_{opt} \in \mathcal{Q}$ such that $q_{opt} = \operatorname{argmax}_{\mathcal{Q}} N(i)$.

The existence of epistemic uncertainties means that $f_{i,y}(\mathbf{x})$ may not be precisely known. We address the case where the mean sea level, $\mu_{s,y}$, is uncertain in the long term due to an uncertain rate of sea level rise. We use the upwelling-diffusion energy balance model MAGICC to generate time series projections of global mean sea level over the 21st century, which are smooth monotonic functions $m(y, \Delta s_{2100})$ of known form, where Δs_{2100} is the predicted mean sea level increase in 2100 relative to present day ($y = 0$), so that $\mu_{s,y} = \mu_{s,0} + m(y, \Delta s_{2100}) + y\delta$, where δ is a constant rate of isostatic subsidence at the site in question. We take MAGICC to be a function of two uncertain quantities, the climate sensitivity T_{2x} and the effective vertical heat diffusivity κ_v, together denoted $\mathbf{z} \doteq (T_{2x}, \kappa_v) : \mathbf{z} \in \mathcal{Z}$. In this context MAGICC therefore can be taken as implementing the function $g : \mathcal{Z} \to \mathbb{R}$ such that $\Delta s_{2100} = g(\mathbf{z})$. The uncertainty in $\mathbf{z}$ is represented as a set $\mathcal{P}(\mathbf{z})$ of probability measures P. If, as is the case here, $\mathcal{P}$ is defined in terms of bounds on a set of density functions, ρ_P, it is then possible to compute the lower probability of mean sea level rise in 2100 exceeding some critical value s_{crit}: $\underline{\Pr}(\Delta s_{2100} > s_{crit}) = \min_{\mathcal{P}} \int_{\mathcal{Z}} I[g(\mathbf{z}) > s_{crit}] \rho_P d\mathbf{z}$ where I is the indicator function. The risk corresponding to a given ρ_P is $R_P(i,y) = \int_{\mathcal{Z}} \int_{\mathcal{X}} D(\mathbf{x}) f_{i,y}(\mathbf{x}|\mu_{s,y}(\mathbf{z})) \rho_P(\mathbf{z}) d\mathbf{x} d\mathbf{z}$. The lower bound on risk in any given year is $\underline{R}(i,y) = \min_{\mathcal{P}} R_P$ and the lower net present value is $\underline{N}_P(i) = \min_{\mathcal{P}} \sum_{y=0}^{t-1} \frac{1}{(1+e)^y} \left(R_P(0,y) - R_P(i,y) - C(i,y)\right)$, and similarly for $\overline{R}(i,y)$ and $\overline{N}(i)$ respectively. We explore a variety of approaches reported in the literature (e.g. [1]) for ordering of decision options given a set of probability measures $\mathcal{P}(\mathbf{z})$, noting that the Net Present Value criterion defined above is a linear combination of expected utilities from each year of the appraisal period.

References

[1] M.J. Schervish, T. Seidenfeld, J.B. Kadane and I. Levi, Extensions of expected utility and some limitations of pairwise comparisons, in *ISIPTA'03 Proc. 3rd Int. Conf. Imprecise Probabilities and Their Applications*, J.-M. Bernard, T. Seidenfeld and M. Zaffalon (eds.), Carlton Scientific, 2003: 496-510.