

Software for Generalized Bayesian Inference

An object-oriented R implementation of generalized iLUCK-models

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Generalized Bayesian Inference – General Idea

Bayesian Inference on some parameter θ :

prior knowledge on θ + data x → updated knowledge on θ

prior distribution $p(\theta)$ + likelihood $f(x | \theta)$ → posterior distribution $p(\theta | x)$

set of priors + likelihood → **set of** posteriors

Tractability: use **conjugate** priors, i.e.

choose $p(\theta)$ such that $p(\theta | x)$ is from the same parametric class

→ update only parameters!

LUCK-models: Single Conjugate Prior

$X \stackrel{iid}{\sim}$ linear, canonical exponential family, i.e.

$$p(x | \theta) \propto \exp \{ \langle \psi, \tau(x) \rangle - n \mathbf{b}(\psi) \} \quad \left[\psi \text{ transformation of } \theta \right]$$

→ conjugate prior:

$$p(\theta) \propto \exp \{ n^{(0)} [\langle \psi, \mathbf{y}^{(0)} \rangle - \mathbf{b}(\psi)] \}$$

→ (conjugate) posterior:

$$p(\theta | x) \propto \exp \{ n^{(1)} [\langle \psi, \mathbf{y}^{(1)} \rangle - \mathbf{b}(\psi)] \}$$

$$\mathbf{y}^{(1)} = \frac{n^{(0)} \mathbf{y}^{(0)} + \tau(x)}{n^{(0)} + n} \quad \text{and} \quad n^{(1)} = n^{(0)} + n.$$

Interpretation of $y^{(0)}$ and $n^{(0)}$

$y^{(0)}$: “main prior parameter”

- ▶ for samples from a $N(\mu, 1)$, $p(\mu)$ is a $N(y^{(0)}, \frac{1}{n^{(0)}})$
- ▶ for samples from a $M(\theta)$, $p(\theta)$ is a $\text{Dir}(n^{(0)}, y^{(0)})$
($y_j^{(0)} = t_j \hat{=}$ prior probability for class j , $n^{(0)} = s$)

$n^{(0)}$: “prior strength” or “pseudocounts”

with $\tilde{\tau}(x) =: \frac{1}{n} \tau(x)$: $\left[\tau(x) = \sum_{i=1}^n \tau(x_i) \right]$

$$y^{(1)} = \frac{n^{(0)}}{n^{(0)} + n} \cdot y^{(0)} + \frac{n}{n^{(0)} + n} \cdot \tilde{\tau}(x).$$

sets of LUCK-models – iLUCK-models

iLUCK-model:

vary $y^{(0)}$ in $\mathcal{Y}^{(0)}$ [$\mathcal{Y}^{(0)}$ convex],

i.e. allow for ambiguity on the main prior parameter

→ prior credal set contains *all finite convex mixtures* of $p(\theta)$ s
with $y^{(0)} \in \mathcal{Y}^{(0)}$

→ posterior credal set easy to calculate:
all finite convex mixtures of $p(\theta | x)$ s with

$$y^{(1)} \in \mathcal{Y}^{(1)} = \frac{n^{(0)}}{n^{(0)} + n} \cdot y^{(0)} + \frac{n}{n^{(0)} + n} \cdot \tilde{\tau}(x)$$



unfavourable behavior in case of prior–data conflict!



sets of LUCK-models – Generalized iLUCK-models

generalized iLUCK-model:

vary $y^{(0)}$ in $\mathcal{Y}^{(0)}$ **and** $n^{(0)}$ in $\mathcal{N}^{(0)}$, i.e. weigh prior information $\mathcal{Y}^{(0)}$ and sample information $\tilde{\tau}(x)$ more flexible in

$$y^{(1)} = \frac{n^{(0)}}{n^{(0)} + n} \cdot y^{(0)} + \frac{n}{n^{(0)} + n} \cdot \tilde{\tau}(x)$$

- prior credal set contains *all finite convex mixtures* of $p(\theta)$ s with $y^{(0)} \in \mathcal{Y}^{(0)}$ **and** $n^{(0)} \in \mathcal{N}^{(0)}$
- posterior credal set still quite easy to calculate: *all finite convex mixtures* of $p(\theta | x)$ s with

$$\left\{ \left(n^{(1)}, y^{(1)} \right) \mid n^{(1)} = n^{(0)} + n, y^{(1)} = \frac{n^{(0)} y^{(0)} + \tau(x)}{n^{(0)} + n}, n^{(0)} \in \mathcal{N}^{(0)}, y^{(0)} \in \mathcal{Y}^{(0)} \right\}$$

The R project for Statistical Computing

- ▶ not just a (statistical) software package, rather a full-grown programming language
- ▶ open source implementation of the (award-winning) S language
- ▶ extremely widespread in university research (reference implementation of new methods are often in R)
- ▶ extensions providing additional functionality can be made readily available as “packages”
- ▶ can be linked with \LaTeX (package Sweave)
- ▶ can be used as imperative or as object-oriented language

Imperative vs. Object-oriented Programming

imperative: do this, then that

➡ functions (on arguments)

object-oriented: create 'objects', do things with them

➡ blueprints for objects called 'classes'

objects created according to a blueprint are called an 'instance'

example:

banking company administrating their customers' accounts

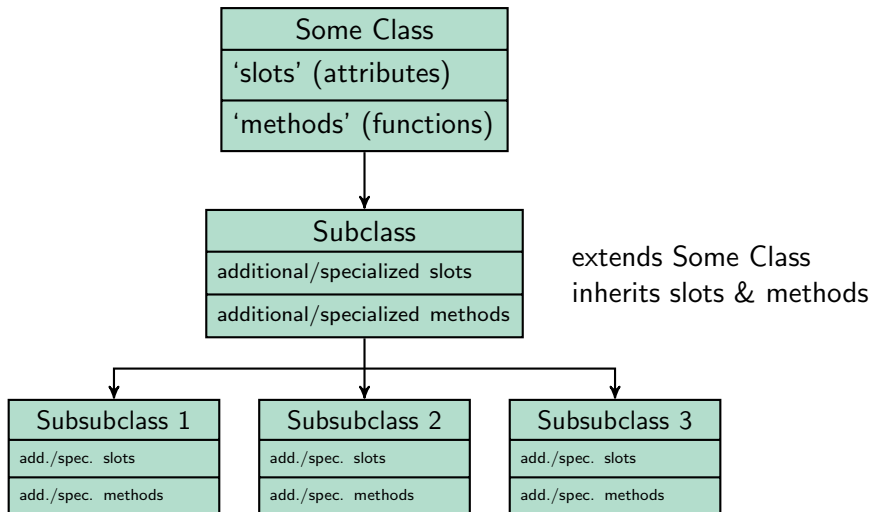
class: `BankAccount`

instances: bank account for customer A

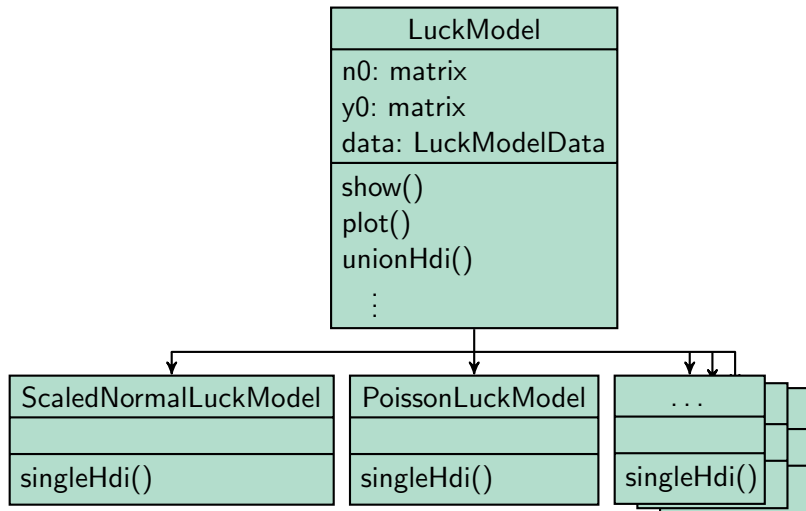
 bank account for customer B

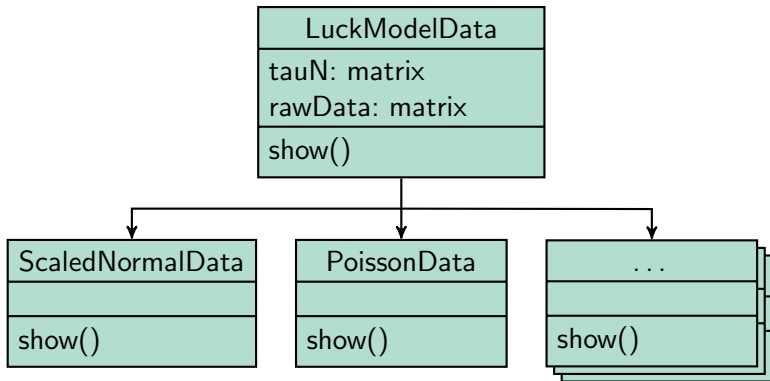
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Object-oriented Programming: Class hierarchies







Implementation – Class Structure





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