NONPARAMETRIC PREDICTIVE INFERENCE FOR ACCEPTANCE SAMPLING WITH DESTRUCTIVE TESTS

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ABSTRACT

An important scenario in quality and reliability applications is acceptance sampling, where items from a production process are tested in order to decide on whether or not to accept a batch of items. A specific form of such testing appears when the test result is simply whether or not a tested unit functions, which is known as 'attribute acceptance sampling'. This paper considers destructive testing, meaning that the tested unit cannot be used again, and corresponding attribute acceptance sampling is considered from nonparametric predictive perspective. It is considered what can be derived in this theory, with only few assumptions used, and with inferences in terms of lower and upper probabilities for the event that the batch will satisfy a suitable reliability criterion in the process after testing

INTRODUCTION

Lower and upper probability, also called 'imprecise probability' or 'interval probability' are attractive approach for prediction. These lower and upper probabilities followed from an assumed underlying latent variable model, with future outcomes of random quantities related to data by Hill's assumption $A_{(n)}$ [3] and are part of a wider statistical methodology called `Nonparametric Predictive Inference' (NPI) which has been developed by Coolen [1].
NPI approach can be used for prediction in case of vague prior knowledge of a probability distribution.

NPI FOR BERNOULLI QUANTITIES

Suppose a sequence of *n+m* exchangeable Bernoulli trials, which are either `good' (`functioning';`success') or `bad' (`not functioning'; `failure') and can be tested in a perfect manner. Let

Y_{n+1}^{n+m} is the random total number of good items out of the *m* items that are produced but that are not tested.

Y_1^n is the number of successes in first *n* items. Then the NPI upper and lower probabilities are

$$\overline{P}(Y_{n+1}^{n+m} \in R_t \mid Y_1^n = s) = \binom{n+m}{n}^{-1} \sum_{j=1}^{t} \left[\binom{s+r_j}{s} - \binom{s+r_j-1}{s}\right]\binom{n-s+m-r_j}{n-s}$$

$$\underline{P}(Y_{n+1}^{n+m} \in R_t \mid Y_1^n = s) = 1 - \overline{P}(Y_{n+1}^{n+m} \in R_t^c \mid Y_1^n = s)$$

Where $R_t = \{r_1, r_2, \ldots, r_t\}$ with $1 \le t \le m+1$ and $0 \le r_1 \le r_2 \le \ldots \le r_t \le m$
.
That for this application we extended it to the following event

$(Y_{n+1}^{n+m} \ge r \mid Y_1^n \ge s)$

$$\underline{P}(Y_{n+1}^{n+m} \ge r \mid Y_1^n \ge s) = \binom{n+m}{m}^{-1} \sum_{j=1}^{t} \left[\binom{s-1+j}{j} - \binom{n-s+m-j}{m-j}\right]$$

$$\overline{P}(Y_{n+1}^{n+m} \ge r \mid Y_1^n \ge s) = 1$$

ACCEPTANCE SAMPLING

Suppose that one wishes to produce *m>0* items for delivery to the market, and in addition one also produces the number *n* required to be tested (so one would actually produce *n+m* items). Assume that tested items cannot be used any further (Destructive Testing). On the basis of the outcome of this test, one wishes to decide on whether or not to 'accept' the *m* further items produced. Let us suppose that one chooses the following criterion,

$$\underline{P}(Y_{n+1}^{n+m} \ge r \mid Y_1^n \ge s) \ge p$$

For some pre-determined m, and for r and p chosen in line with a quality requirement. The main task of inferential methods for acceptance sampling is to determine pairs *(n; s)* for which this criterion is satisfied.

EXAMPLE FOR THE DESTRUCTIVE TEST

Suppose one requires *m = 10* items for future use, and wishes to test n other items. The table below shows that these inferences are strongly influenced by choice of *p*

. •Minimum required *n* for different values of *p*.

p	*s=n, r=10*	*s=n-1, r=10*	*s=n, r=9*
0.50	10	25	4
0.70	24	52	8
0.80	40	86	12
0.90	90	186	21
0.95	190	386	33
0.98	490	986	58
0.99	990	1986	86

TWO-STAGE ACCEPTANCE SAMPLING

Suppose that, it is needed to decide on a suitable number of items to be tested in the first stage, with the knowledge that, if the test results do not clearly indicate a final decision (accept or reject the batch of *m* items), one can test a further n_2 items before making the final decision. A natural criterion for accepting the batch in the first stage is

$$\underline{P}(Y_{n_1+1}^{n_1+m} \ge r \mid Y_1^{n_1} = s_1) \ge p_1$$

where

$$\underline{P}(Y_{n_1+1}^{n_1+m} \ge r \mid Y_1^{n_1} \ge s_1) = \underline{P}(Y_{n_1+1}^{n_1+m} \ge r \mid Y_1^{n_1} = s_1)$$

After the first stage of testing, the decision to immediately reject the batch after the first stage of testing is based on

$$\overline{P}(Y_{n_1+1}^{n_1+m} \ge r \mid Y_1^{n_1} \le s_1) \le q_1$$

If $\underline{P} < p_1$ and $\overline{P} > q_1$ the second sample should be tested. In this case, we would accept the whole batch if

$$\underline{P}(Y_{n_1+n_2+1}^{n_1+n_2+m} \ge r \mid Y_1^{n_1+n_2} = s_1 + s_2) \ge p_2$$

Otherwise the batch should be rejected.

TWO-STAGE ACCEPTANCE SAMPLING

Suppose that $p_1 = 0.8$ $q_1 = 0.50$ and $m = 10$. Suppose further that we can get 10 items tested at the first testing stage, and if the results prove to be inconclusive we can test a further 10 items. In this setting, suppose first that all 10 items in stage one of testing functioned successfully. As

$$\underline{P}(Y_{11}^{20} \ge 8 \mid Y_1^{10} = 10) = 0.89 \ge 0.8$$

we accept the batch of 10 further items without further testing. If, instead, there was one item that failed this test

$$\underline{P}(Y_{11}^{20} \ge 8 \mid Y_1^{10} = 9) = 0.71 < 0.8$$

$$\overline{P}(Y_{11}^{20} \ge 8 \mid Y_1^{10} = 9) = 0.89 > 0.5$$

Therefore, the second stage of testing will be used. Suppose that in total 19 out of 20 items tested functioned. This leads to

$$\underline{P}(Y_{21}^{30} \ge 8 \mid Y_1^{20} = 19) = 0.9 \ge 0.80$$

And now the batch will be accepted.