

Solving Decision Trees with Imprecise Probabilities through Linear Programming

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Abstract

Previous algorithms that generate policies in decision trees with imprecise probabilities [1, 7] employ multilinear programming to compute expected values for policies, that is, a program where the objective function involves a summation of products of variables, and the constraints are linear functions defining the set of probabilities. Despite being a well studied field, multilinear programs are still considered to be difficult problems, thus motivating the investigation of easier ways to obtain these values.

We show that for a specific type of decision tree a linear programming scheme can be obtained for this problem by replacing products of conditional probabilities (variables of the problem) by joint probabilities. Hence, instead of a sum of products of variables, we now have a sum of variables, eliminating the complex multilinear characteristic. The number of atomic events grows exponentially with the number of chance nodes in the decision tree (minus the possible repetition of events in some chance nodes). The resulting linear program contains a number of constraints that is linear on the size of the input decision tree, but a number of optimization variables that is exponential on the number of chance nodes. We thus face a situation that is typical of *probabilistic logic* [2, 3, 4], and that has been tackled with column generation [6] and redundancy detection [5, 8] methods.

We restrict ourselves to the column generation method, and investigate a possible formulation for its use in solving decision trees with imprecise probabilities.

Keywords. Decision trees, partial preference, linear programming, column generation.

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