Efficient Computing of a Least Favorable Pair for Two Hypothesis of Probability Intervals

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Abstract

In this contribution to the theory of probability intervals the construction of the risk-function is used to determine a least favorable pair of probabilities.

We pose $\Omega = \{\omega_i \,|\, 1 \leq i \leq n\}$ as finite sample space and H_0 the (feasible) probability interval with the intervals $[L_0(E_i)\,,\,U_0(E_i)]$ for $E_i = \{\omega_i\}$ and $1 \leq i \leq n$ and $\mathcal{M}_0$ the set of probabilities which lie in all these intervals. Let H_1 the (feasible) probability interval with the intervals $[L_1(E_i)\,,\,U_1(E_i)]$ for $E_i = \{\omega_i\}$ and $1 \leq i \leq n$ and $\mathcal{M}_1$ the set of probabilities which lie in all these last introduced intervals.
If $\mathcal{M}_0 \cup \mathcal{M}_1 \neq \emptyset$, a least favorable pair of probabilities $(q_0\,,\,q_1)$ with $q_0 \in \mathcal{M}_0$ and $q_1 \in \mathcal{M}_1$ exists. The risk-function of the Neyman-Pearson-Test of q_0 vs. q_1 is at the same time the risk-function of the Niveau α-Maximintest H_0 vs. H_1 and at the same time the lower convex envelope (called $KUR(X,.)$) of the pointset $X = \{(0,1),(1,0)\} \cap \{(U_0(A), U_1(\neg A)) | \emptyset \neq A \subseteq \Omega\}$ where $U_0(A) = \max_{p \in \mathcal{M}_0} p(A)$ and $U_1(A) = \max_{p \in \mathcal{M}_1} p(A)$.
To find q_0 and q_1 for two hypothesis H_0 and H_1 of disjoint probability intervals, one may construct the lower convex envelope of the points X ($|X| \leq 2^n$) to have the risk-function q_0 vs. q_1.
In order to avoid to determine many points for large n a supplementary proceeding can be choosen:
Take Z as the variable of "possible formular cases": $Z \in \{(I,I),(I,II),(II,I),(II,II)\}$. Regard the values as $(U_0(A), U_1(\neg A))$ lying in a plane with coordinates x and y. For $A \neq \emptyset$ set

$$val_{x\,Z}(A) = \begin{cases} \sum_{E \subseteq A} U_0(E), \; Z = (I,I),(II,I) \\ 1 - \sum_{E \subseteq \neg A} L_0(E), \; Z = (I,II),(II,II) \end{cases} \qquad val_{y\,Z}(A) = \begin{cases} 1 - \sum_{E \subseteq A} L_1(E), \; Z = (I,I),(I,II) \\ \sum_{E \subseteq \neg A} U_1(E), \; Z = (II,I),(II,II) \end{cases}$$

We build for $Z \in \{(I,I),(I,II),(II,I),(II,II)\}$ the four sets
$X_Z = \{(0,1),(1,0)\} \cap \{(val_{x\,Z}(A), val_{y\,Z}(A)) | \emptyset \neq A \subseteq \Omega\}$. For all Z the lower convex envelope of the sets X_Z (called $KUR(X_Z,.)$) has no point below the lower convex envelope of the set X.
Each of the lower convex envelope of $X_{I,I}, X_{I,II}, X_{II,I}, X_{II,II}$ is (via an unified algorithm) constructed by at most n points. The convex lower envelope of X can be constructed (efficiently) by building the lower convex envelope of all these at most four times n resulting points of the construction for each X_Z. Now by the risk-funktion the least favorable pair of probabilities can be calculated. This is a result (to be illustrated in the poster) in [3].

Keywords. probability interval, least favorable pair of probabilities

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