Efficient Computing of a Least Favorable Pair for Two Hypothesis of Probability Intervals

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Abstract

In this contribution to the theory of probability intervals the construction of the risk-function is used to determine a least favorable pair of probabilities.

We pose $\Omega = \{\omega_i | 1 \leq i \leq n\}$ as finite sample space and $H_0$ the (feasible) probability interval with the intervals $[L_0(E_i), U_0(E_i)]$ for $E_i = \{\omega_i\}$ and $1 \leq i \leq n$ and $\mathcal{M}_0$ the set of probabilities which lie in all these intervals. Let $H_1$ the (feasible) probability interval with the intervals $[L_1(E_i), U_1(E_i)]$ for $E_i = \{\omega_i\}$ and $1 \leq i \leq n$ and $\mathcal{M}_1$ the set of probabilities which lie in all these last introduced intervals. If $\mathcal{M}_0 \cup \mathcal{M}_1 \neq \emptyset$, a least favorable pair of probabilities $(q_0, q_1)$ with $q_0 \in \mathcal{M}_0$ and $q_1 \in \mathcal{M}_1$ exists. The risk-function of the Neyman-Pearson-Test of $q_0$ vs. $q_1$ is at the same time the risk-function of the Niveau $\alpha$-Maximintest $H_0$ vs. $H_1$ and at the same time the lower convex envelope (called $\text{KUR}(X,)$) of the pointset $X = \{(0,1), (1,0)\} \cap \{(U_0(A), U_1(\neg A))|0 \neq A \subseteq \Omega\}$ where $U_0(A) = \max_{p \in \mathcal{M}_0} p(A)$ and $U_1(A) = \max_{p \in \mathcal{M}_1} p(A)$. To find $q_0$ and $q_1$ for two hypothesis $H_0$ and $H_1$ of disjoint probability intervals, one may construct the lower convex envelope of the points $X (|X| \leq 2^n)$ to have the risk-function $q_0$ vs. $q_1$. In order to avoid to determine many points for large $n$ a supplementary proceeding can be chosen: Take $Z$ as the variable of "possible formular cases": $Z \in \{(I, I), (I, II), (II, I), (II, II)\}$. Regard the values as $(U_0(A), U_1(\neg A))$ lying in a plane with coordinates $x$ and $y$. For $A \neq \emptyset$ set

$\text{val}_x(Z)(A) = \begin{cases} \sum_{E \subseteq A} U_0(E), & Z = (I, I), (II, I) \\ 1 - \sum_{E \subseteq \neg A} L_0(E), & Z = (I, II), (II, II) \end{cases}$

$\text{val}_y(Z)(A) = \begin{cases} 1 - \sum_{E \subseteq A} L_1(E), & Z = (I, I), (I, II) \\ \sum_{E \subseteq \neg A} U_1(E), & Z = (II, I), (II, II) \end{cases}$

We build for $Z \in \{(I, I), (I, II), (II, I), (II, II)\}$ the four sets $X_Z = \{(0,1), (1,0)\} \cap \{(\text{val}_x(Z), \text{val}_y(Z))|0 \neq A \subseteq \Omega\}$. For all $Z$ the lower convex envelope of the sets $X_Z$ (called $\text{KUR}(X_Z,)$) has no point below the lower convex envelope of the set $X$. Each of the lower convex envelope of $X_{I,I}, X_{I,II}, X_{II,I}, X_{II,II}$ is (via an unified algorithm) constructed by at most $n$ points. The convex lower envelope of $X$ can be constructed (efficiently) by building the lower convex envelope of all these at most times $n$ resulting points of the construction for each $X_Z$. Now by the risk-funktion the least favorable pair of probabilities can be calculated. This is a result (to be illustrated in the poster) in [3].

Keywords. probability interval, least favorable pair of probabilities

References