Some Incoherencies Resulting from Minimal Information Methods

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Abstract

The technique of minimizing information (henceforth, infomin) has been commonly employed as a general method for updating one's prior probabilities. In the past, infomin updating has been criticized on the grounds that the updating behavior it recommends cannot be represented within a higher-order Bayesian framework. We argue that, in a wide class of cases, infomin can be faulted on more fundamental grounds without presupposing that an agent is always committed to a single, determinate higher-order probability distribution.

In general, the evidence on which one updates is given as a constraint C_λ, where λ is the value of an unknown parameter (a real number or a vector of reals) which takes its value from the set Λ. Let U be an arbitrary updating method. For a single probability function Pr and a constraint C_λ, updating Pr by means of U, results in a posterior probability Pr_λ which satisfies C_λ. We describe an event A as *always decreasing under* U with respect to the family of constraints $\{C_\lambda\}_{\lambda\in\Lambda}$ and the probability Pr, if for all $\lambda \in \Lambda$ either $Pr_\lambda = Pr$ or $Pr_\lambda(A) < Pr(A)$. A is *always increasing under* U if for all $\alpha \in \Lambda$ either $Pr_\alpha = Pr$ or $Pr_\alpha(A) > Pr(A)$. For a given probability Pr, U is *deceptive over* A with respect to the family of constraints $\{C_\lambda\}_{\lambda\in\Lambda}$, if A is either always decreasing or always increasing under U. We call U deceptive if it is deceptive over some A.

We argue that non-deceptiveness is a necessary condition for the rational coherence of an updating method. Our argument generalizes in a natural way to the updating of imprecise probabilities (represented as a collection of priors), and imprecise constraints (represented as a subset of $\{C_\lambda\}_{\lambda\in\Lambda}$). However, it cannot be applied to the updating of indeterminate probabilities. We show that the existence of a higher-order probability distribution that 'supports' an updating method (in a sense to be made precise), guarantees its non-deceptiveness.

If $B \subset A$ and $0 < Pr(B) < Pr(A) < 1$, then infomin updating is deceptive over A with respect to Pr and the family of constraints $\{P(B|A) = \lambda\}_{\lambda\in[0,1]}$. In the case in which the constraints are of the form $E(X) = \lambda$ for some real-valued random variable X, the situation is more complicated. Infomin updating of the uniform prior is deceptive for some random variables, but not for others. Since all such updating lacks higher-order support, this shows that higher-order support is a strictly stronger condition than non-deceptiveness.

The use of infomin for updating the uniform prior should be clearly distinguished from its use in selecting a prior. Failure to observe this distinction has led, in the past, to misplaced criticisms. However, even when infomin is used for prior selection, the family of constraints should not be conditional-probability constraints if we wish to avoid certain counterintuitive consequences. Moreover, in the case of expected-value constraints, the random variable cannot be arbitrary. It must have significance in the context of an accepted background of theory if the chosen prior is to be coherently interpreted as a subjective probability.

Keywords. Shannon information, Kullback-Leibler divergence, higher-order probability.

References

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