1 Context
Agent faced with uncertainty; Possibility space $\Omega$, e.g., set of experimental outcomes; Gamble $\text{g}$ valued functions $f$ on $\Omega$, interest in a linear space of gambles $\mathbb{C}$.
Example format used for illustrations: $\Omega = \{a, b, c\}$, and $\mathbb{C}$ is the linear space of all gambles on $\Omega$.

2 Accepting & rejecting
The agent gives an assessment $A$ by making statements about gambles $f$: Accepting $(\square)$ implies a commitment: if $(\square)$ outcome $\Omega_0$ is determined, then $\text{g}(\Omega_0)$ is accepted. Rejecting $(\Box)$ can be understood as $\text{g}(\Omega_0)$ is unacceptable.

3 Deductive closure
We assume gamble payoffs are expressed in a linear precise utility. This implies:
Positive scaling if $\text{g}$ is acceptable, then all gambles in the ray $\text{g} = t \cdot \text{g}$ are acceptable.
Combination if $\text{g}_1$ and $\text{g}_2$ are acceptable, then $\text{g}_1 + \text{g}_2$ is acceptable.
So we can extend an assessment $A$.

4 No limbo
Let $\text{D} \in \mathbb{D}$, then without increasing confusion $\text{g}$ can be accepted if and only if $\text{g} \in (\text{D}, \ominus)$. But $\text{D}$ is the set of unresolvable gambles.

5 Models properties
Given a model $M$ in $\mathbb{M}$, then $\text{M} \models \text{M}_0 \iff (\text{M}_0 \models \text{M})$.

6 Set operations
\begin{align*}
\text{Assessment union} \quad & \cap \quad \text{centre-point set union;} \\
\text{Assessment intersection} \quad & \cap \quad \text{componensite set intersection;}
\end{align*}
The sets $A$, $B$, $D$, and $M$ are closed under arbitrary non-empty intersections, but $M$ is not, as is attested by the counterexample below:

7 Order-theoretic results
The ‘less committal than or equal’ relation $\subseteq$ engenders a partial ordering of the assessments.

8 Dominating models
The agent specifies an assessment $A$; it is an element of $\mathbb{A}$, can be extended to a model without confusion: $M = \{\text{M} \models \text{M}_0 \mid \text{M}_0 \models \text{M} \}$.

9 Smallest models
Some a priori assumptions can be captured by positing a smallest model $S$ in $\mathbb{M}$ that replaces $\mathbb{M}$ as we work in $\mathbb{M}_0$ instead of $\mathbb{M}$.

10 Universal a priori assumption
Given the commitments implied by accepting gambles, there is a one assumption we judge reasonable to posit.

Indifference to status quo $\cap \Omega$ is: 

\begin{align*}
\text{If and only if} & \quad \text{if and only if} \\
\text{Deductive closable} & \quad \text{Deductive closable}.
\end{align*}

11 Partitions
All models $M$ in $\mathbb{M}$ partition gamble space $\Omega$ into nine elements—some possibly empty—depending on whether a gamble and its negation are acceptable, disapprovable, or neither.

12 Basic gamble relations
Fix a model $M$ in $\mathbb{M}_0$, then we can define the following relations between gambles $\text{g}$ and $\text{g}'$ in $\Omega$:

\begin{align*}
& \text{if is accepted in exchange for} \quad \text{if is accepted in exchange for} \\
& \text{if is accepted in exchange for} \quad \text{if is accepted in exchange for} \\
& \text{if is accepted in exchange for} \quad \text{if is accepted in exchange for}.
\end{align*}
The ‘axioms’ defining $M_0$—no confusion, deductive closure, no limbo, and indifference to status quo—can then be reformulated:

\begin{align*}
\text{No confusion} & \quad \text{if is accepted in exchange for} \\
\text{Reversibility} & \quad \text{if is accepted in exchange for} \\
\text{Transitivity} & \quad \text{if is accepted in exchange for}.
\end{align*}

\text{Mixture independence for all } \text{c} \in (0, 1) \text{ and } \text{c} \not= 1:

\begin{align*}
\text{if is accepted in exchange for} & \quad \text{if is accepted in exchange for} \\
\text{if is accepted in exchange for} \quad \text{if is accepted in exchange for} \\
\text{if is accepted in exchange for} \quad \text{if is accepted in exchange for}.
\end{align*}

13 Derived gamble relations
A number of other useful gamble relations follow from the basic ones:

\begin{align*}
\text{Indifference between } & \text{and } \text{if is accepted in exchange for} \\
\text{and } \text{if is accepted in exchange for} \quad \text{and } \text{if is accepted in exchange for} \\
\text{and } \text{if is accepted in exchange for} \quad \text{and } \text{if is accepted in exchange for}.
\end{align*}

14 Smallest models
Association the relations $\subseteq$ and $\subseteq$ with the smallest model $S$ in $\mathbb{M}_0$, then a necessary condition for coherence is:

\begin{align*}
\text{Monotonicity} & \quad \text{if is accepted in exchange for} \\
\text{if is accepted in exchange for} & \quad \text{if is accepted in exchange for}.
\end{align*}

15 Desirability
Strict preference desirability is a simplification with $M$ in $\mathbb{M}_0$ such that $M_0 \models \text{M}_0$. $\subseteq$,

16 Comparison with the literature
Forms of what we call strict preference desirability have been gotten the most attention in the literature:

Smith (1961, §14) talks about ‘exchange vectors’ (finite $\mathbb{O}$), works with $S = \mathbb{M}_0$, and imposes that $M_0$ is open.

Seidenfeld et al. (1990, §III) talk about ‘favorable’ gambles (finite $\mathbb{O}$) and work with $S = \mathbb{M}_0$. Walley (1991, §3.7) discusses ‘strictly desirable’ gambles, works with $S = \mathbb{M}_0$, and imposes an openness axiom $M_0 \subseteq M \subseteq \mathbb{M}_0$. Walley (2000, §6) drops the openness axiom and advocates a desirability framework with $S = \mathbb{M}_0$.

De Cooman & Shaferly (2009–2011) build on this, but are the first there will be a nontrivial $S$, i.e., the gambles expressing expressibility.

Ocurrences of the nonstrict case are also important:

Williams (1974) talks about ‘acceptable bets’ and works with $S$ defined by $S = \{c \in \mathbb{O} \mid c > 0\}$, so there is no default indifference to status quo.

Walley (1991, §3.2) discusses almost desirable gambles, works with $S = \{c \in \mathbb{O} \mid c > 0\}$, and imposes a closure axiom $S = S_0 \subseteq M \subseteq \mathbb{M}_0$. Walley (1991, 3.6.5) talks about ‘realistic desirable’ gambles, works with $S = \{c \in \mathbb{O} \mid c > 0\}$.