Linear programming under vacuous and possibilistic uncertainty

Keivan Shariatmadar, Erik Quaeghebeur & Gert de Cooman
SYSTeMS Research Group, Ghent University
{Keivan.Shariatmadar,Erik.Quaeghebeur,Gert.deCooman}@UGent.be

Abstract

Consider the following (standard) linear programming problem: maximise a real-valued linear function $C^T x$ defined for optimisation variables $x$ in $\mathbb{R}^n$ that have to satisfy the constraints $Ax \leq B$, $x \geq 0$, where the matrices $A$, $B$, and $C$ are independent random variables that take values $a$, $b$, and $c$ in $\mathbb{R}^{m \times n}$, $\mathbb{R}^m$ and $\mathbb{R}^n$, respectively. Using an approach we developed in previous work, the problem is first reduced to a constrained optimisation problem (co-problem) from which the uncertainties present in the description of the constraint are eliminated. The goal is to derive efficient solution techniques for this resulting co-problem.

We investigate what results can be obtained for two types of uncertainty models for the random variables $A$, $B$, and $C$ – vacuous previsions and possibility distributions [see, e.g., 1, 5] – and for two different optimality criteria – maximinity and maximality [see, e.g., 4]. In our poster, we will present the problem description and show illustrated solutions for the most interesting cases we have investigated. We consider three variants of our problem: (i) when there is no uncertainty about $C$ (this exactly fits the approach in [3]), (ii) when there is no uncertainty about $B$, which reduces to variant (i) when considering the dual, and (iii) the general case, which we can convert to the following problem: maximise the real value $\lambda$ such that $Ax \leq B$, $C^T x \geq \lambda$ and $x \geq 0$, and which is the subject of current research. We here focus on variant (i).

For the different cases we studied, the co-problem and solution techniques derived are:

- **Vacuous model relative to a set $A \subseteq \mathbb{R}^{m \times n} \times \mathbb{R}^m$:**
  - The maximin solution $x_m$ can be found by solving the linear programming problem $\arg \max_{x \in \mathbb{R}^n} \{c^T x \mid L \} P_\pi (Ax \leq B)$ where $L$ is a penalty for violating the constraints. When the possibility distribution $\pi$ is unimodal then $(c^T x - L) P_\pi (Ax \leq B)$ is unimodal too because of the linearity of the objective function, which allows us to find the maximin solution using a bisection method in which each step a linear programming problem must be solved.
  - We have not yet found an efficient way to calculate the maximal solutions. We can approximate the solutions when $m$ and $n$ are small enough.

**Keywords.** linear programming, maximinity, maximality, vacuous prevision, possibility distribution.

References