The Extension Principle applied to Stochastic Differential Equations

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Abstract

Stochastic differential equations (SDEs) have become a powerful tool to model processes arising in nature, engineering or economic sciences which are not deterministic, but subject to random fluctuation. But even since random disturbances on the system can be described by a stochastic differential term, there remains a "structural uncertainty", fluctuations of the predicted behaviour of the system due to unknown or uncertain values of model parameters. Since the model parameters determine the stability and the equilibria of the model system and, hence, its long run behaviour, we see the need of modelling model uncertainty. A fuzzy approach, i.e. modellig the uncertainty by fuzzy methods, seems to be an approriate way.

In the last years much effort has been made to investigate *set-valued and fuzzy-set-valued stochastic differential equations* ([1, 2]). But as it turned out ([3]), the approaches for set-valued SDEs can handle multivalued drift but just singe-valued diffusion coefficients. Therefore, uncertainty in diffusion parameters can not be modelled by set-valued SDEs.

We have developed an approach based on the extension principle, where the uncertainty in the data of the SDE (initial value and coefficients) is understood as a fuzzy set of random variables and coefficient functions. This allows to model more advanced types of uncertainty than in approaches for SDEs with fuzzy set-valued coefficients, e.g. uncertain but time constant or slowly changing model parameters, dependencies between drift and the diffusion term (by depending on the same model parameters) et cetera.

The fuzzy set of data, which is identified with a fuzzy set of SDEs (called *fuzzy SDE*), is further mapped to a fuzzy set of cádlág processes which solve the corresponding SDEs. By defining appropriate metric spaces for the coefficients and the initial values, the extension principle allows to maintain properties of the uncertainty structure of the data like boundness and compactness through the continuity of the mapping. We therefore state a locally Lipschitz-continuous dependence of the solution of an SDE w.r.t. the initial value and the coefficients. Moreover, by applying the extension principle rigorously we can develop a theoretical framework to work with fuzzy sets of processes (called *fuzzy processes*) and martingales (*fuzzy martingales*). In particular, it is possible to define charateristics like estimation, variance or even probabilites of certain events for the fuzzy solution of our approach. The case of uncertain model parameters modelled by a fuzzy set of (time-depended) parameters can be considered as a special case of a fuzzy SDE.

Keywords. Stochastic differential equations, fuzzy theory, extension principle, parameter uncertainty

References

[1] Kim, Jai Heui: *On Fuzzy Stochastic Differential Equations* in: *J. Korean Math. Soc.* **42** No. 1, pp. 153-169 (2005).

[2] Zhang, Jinping et al.: *On the solutions of set-valued stochastic differential equations in M-type 2 Banach spaces* in: *Tohoku Math. J.* **61**, pp. 417-440 (2009).

[3] Ogura, Yukio: *On Stochastic Differential Equations with Set Coefficients and the Black-Scholes Model* in: *Proceedings of the Eighth International Conference on Intelligent Technologies*, pp. 300-304 (2007).