Credal Compositional Models and Credal Networks

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Credal networks

A credal network over $X_N$ is (in analogy to Bayesian networks) a pair $(G, (P_1, \ldots, P_k))$ such that, for any $i = 1, \ldots, k$, $(G, P_i)$ is a Bayesian network over $X_N$ i.e., each $P_i$ is a system of conditional probability distributions forming the joint distribution of $X_N$, $P_i(X_N)$. The resulting model is a credal set, which is the convex hull of the Bayesian networks, i.e.,

$$\text{CH}(P_1(X_N), \ldots, P_k(X_N)).$$

Separately specified credal networks

A separately specified credal network over $X_N$ is a pair $(G, \mathbf{M})$, where $\mathbf{M}$ is a set of conditional credal sets $M_i(X_i|\pi(X_i))$ for each $X_i \in X_N$, and $\pi(X_i)$ denotes the set of parent variables of $X_i$. Here the overall model is in analogy to Bayesian networks, obtained as a strong extension of the $M_i(X_i|\pi(X_i))$, $i \in N$.

Relation among different models

Let us denote by $\text{CN}(X_N)$, $\text{SCN}(X_N)$ and $\text{CM}(X_N)$ the class of all credal networks over $X_N$, the class of all separately specified credal networks over $X_N$ and the class of compositional models over $X_N$, respectively.

For any $X_N$

$$\text{SCN}(X_N) \subset \text{CM}(X_N) \subset \text{CN}(X_N).$$

From perfect sequence to credal network

Having a perfect sequence $M_1, M_2, \ldots, M_n$, $M_1$ being a credal set describing $X_1$, we first order all of the variables for which at least one of the credal sets $M_i$ is defined in such a way that we first order (in an arbitrary way) variables for which $M_1$ is defined, then variables from $M_2$ that are not contained in $M_1$, etc. Finally we have

$$(X_1, X_2, \ldots, X_N) = \{X_i\}_{i \in K_1 \cup \ldots \cup K_n},$$

Then we get a graph of the constructed evidential network in the following way: 1. the nodes are all the variables $X_1, X_2, \ldots, X_N$, 2. there is an edge $(X_i \rightarrow X_j)$ if there exists a credal set $M_i$ such that both $i, j \in K_l$, $i \neq j$, and either $i \in K_l \cup \ldots \cup K_{l-1}$ or $j \in K_l \cup \ldots \cup K_{l-1}$ or $i < j$. Having the structure of the credal network, i.e., graph $G$, one can obtain the systems of conditional probability distributions from corresponding perfect sequences of probability distributions.