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Message from the editor

This issue of the SIPTA newsletter opens with a piece on applications of imprecise probabilities in finance. The text contains material presented by Prof. Paolo Vicig in his tutorial for the Fourth Int. Symp. on Imprecise Probabilities and Their Applications (ISIPTA '05). You will also find a brief summary of the events at ISIPTA '05; of special important is the summary of decisions taken at the SIPTA meeting that occurred during the symposium. We thank Marco Zaffalon, SIPTA's secretary, for preparing the summary of the SIPTA meeting.

We also have announcements for the Second SIPTA School on Imprecise Probabilities, for the Third Int. Conf. on Soft Methods in Probability and Statistics, and for special issues of the Int. Journal on Approximate Reasoning (IJAR). We note that IJAR has recently gone through major changes; there are now several areas of specific interest to the journal — among them imprecise probabilities, whose Area Editor is Marco Zaffalon.

Finally, in the Software section you will find two valuable tools: the CkC package, distributed by Capotorti and Vantaggi, and the rcdd package, distributed by Geyer, Lazar and Meeden.

If you have contributions to make to this Newsletter, or if you know of any event or publication that should be of interest to members of SIPTA, please let me know (send a message to fgcozman@usp.br).

Cheers!

Fabio G. Cozman

Imprecise Probabilities and Financial Risk Measurement

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The following material is based on a tutorial delivered by the author at ISIPTA '05, Pittsburgh, PA, USA, on 20 July 2005.

Although financial risk measurement is a largely investigated research area, its relationship with imprecise probabilities has been mostly overlooked. However risk measures can be viewed as instances of upper (or lower) previsions, thus letting us apply the theory of imprecise previsions to them.

After a presentation of some well known risk measures (including Value-at-Risk or *VaR*, coherent and convex risk measures), we show how their definitions can be generalized and discuss their consistency properties.

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Thus, for instance, *VaR* may or may not avoid sure loss, and conditions for this can be derived. This analysis also makes us consider a large class of imprecise previsions, which we termed convex previsions, generalizing convex risk measures and other uncertainty models.

Measures for conditional risks can further be introduced by extending this class to conditional convex previsions. Finally, we discuss the role of some important notions in the theory of imprecise probabilities when applied to risk measurement.

Risk Measurement without Imprecise Probabilities

A basic problem in financial risk measurement is that of measuring how *risky* a given random variable X is. In practice X will be a certain debenture, a share, an index, a portfolio or a subportfolio, and so on.

Although various instruments can theoretically be employed to tackle this problem, for instance loss functions, practitioners tend to favour *risk measures*, also because of their conceptual simplicity. In fact, the risk measure $\rho(X)$ for X is just a real number which should summarize the evaluation about the riskiness of X. It has a direct *operational interpretation*: when positive, it should measure the *risk capital* which the owner of X should allocate to face possible losses arising from X. Multiples of some risk measure are in fact used by banks and other companies to determine (daily or weekly) the level of the reserve funds covering risks related to their portfolios. When negative, $\rho(X)$ represents the amount of money which could be subtracted from X, keeping the resulting random variable acceptable, or in other words *desirable*.

More generally, one might consider an arbitrary set D of random variables, and associate a real number $\rho(X)$ to each of them. The risk measure ρ is then a real function with domain D.

Also, the outcome of each X in D will usually be determined only at a certain future time t_X (generally random, but we assume it is nonrandom here). Hence, what is under evaluation is actually a discounted value for X, i.e. X multiplied for a convenient discounting factor r. To make things simpler, we assume $r = 1$. This is not restrictive for the coming theory, and corresponds to a situation when the gap between the evaluation time and t_X is negligible, or when the discounting factor is anyway close to 1.

Clearly, the problem of choosing a risk measure is a delicate one, and it seems difficult to find proposals free of any shortcoming and criticism. I present here some solutions, among those currently most used or investigated, but will not include other important kinds of risk measures.

Probably, *Value-at-Risk* or VaR is nowadays the most widespread risk measure. Following [2], it is defined in this way:

Definition 1 *Let X be a random variable, whose probability distribution is P. The number q is an* α-quantile *for X if*

$$P(X < q) \leq \alpha \leq P(X \leq q). \tag{1}$$

Define then

$$q_\alpha^+(X) = \inf\{x : P(X \leq x) > \alpha\} \tag{2}$$

$$VaR_\alpha(X) = -q_\alpha^+(X). \tag{3}$$

Hence, VaR_α is a quantile-based measure. As such, a sufficiently reliable estimate of the distribution function of any X in D must be available to use it, and this may already be difficult in certain situations.

Criticism on VaR has focused also on other points:

- VaR is nearly uninformative about the values of X smaller than the threshold q_α^+, only letting us know that as a whole their probability is bounded above by the chosen α. In particular, it tells us nothing about the maximum loss X may cause. Clearly, given P, we obtain more prudent risk evaluations from VaR_α by lowering α, so α is usually fixed *a priori* at a level considered sufficiently low, for instance $\alpha = 0.01$.

- VaR is concerned with potential losses, but disregards possibly high gains. For instance, suppose X and Y are such that $VaR_\alpha(X) = VaR_\alpha(Y)$, but X guarantees a gain of one million dollars or more with a probability of 0.1, while the maximum gain from Y is certain to be considerably smaller. X and Y could not be distinguished if VaR were the only tool for comparing them.

- VaR is not necessarily subadditive, i.e. there exist some D and $X, Y \in D$ such that

$VaR_\alpha(X+Y) > VaR_\alpha(X) + VaR_\alpha(Y)$. Subadditivity is an often desirable property: for instance, a firm willing to reserve as little money as possible to cover its portfolio risks and adopting a non-subadditive risk measure might find it useful to split, possibly artificially, the portfolio into two or more subportfolios. Another strong argument is that the risk of the sum should be not greater than the sum of the risks, because of diversification of investments.

Recently, a new family of risk measures, *coherent risk measures*, was introduced as an alternative to VaR in a series of papers (among these, [1, 2, 3]) by Artzner, Delbaen, Eber and Heath. Coherent risk measures were defined by a set of axioms on a linear space. In the version of [2], the definition is:

Definition 2 *Let $\mathcal{L}$ be a linear space of random variables which contains real constants. A mapping ρ from $\mathcal{L}$ into $\mathbb{R}$ is a* coherent risk measure *iff it satisfies the following axioms:*

T) $\forall\, X \in \mathcal{L}, \forall\, \alpha \in \mathbb{R}$, $\rho(X+\alpha) = \rho(X) + \alpha$ *(translation invariance)*

PH) $\forall\, X \in \mathcal{L}, \forall\, \lambda \geq 0$, $\rho(\lambda X) = \lambda\rho(X)$ *(positive homogeneity)*

M) $\forall\, X, Y \in \mathcal{L}$, *if* $X \leq Y$ *then* $\rho(Y) \leq \rho(X)$ *(monotonicity)*

S) $\forall\, X, Y \in \mathcal{L}$, $\rho(X+Y) \leq \rho(X) + \rho(Y)$ *(subadditivity).*

Actually, all X in $\mathcal{L}$ are simple in [2], i.e. they may assume only finitely many distinct values, but this requirement is inessential for the sequel and is dropped here.

Coherent risk measures are subadditive, and also positively homogeneous. Axiom *(PH)* has been considered the least convincing one in Definition 2. In fact, it could reasonably be $\rho(\lambda X) > \lambda\rho(X)$, for some $\lambda > 1$: holding very large amounts of a financial investment might be proportionally much more risky than holding a more limited quantity, for various reasons, including *liquidity risks* (we might be forced to allow a significant discount to the buyer(s) when wishing to sell quickly large quantities of a certain investment).

Also for these reasons, a generalization of the notion of coherent risk measure (according to [1, 2]) was suggested by Föllmer and Schied in [5, 6]. They defined convex risk measures on linear spaces of random variables by substituting axioms (S) and (PH) in Definition 2 with the *convexity axiom*

$$\rho(\lambda X + (1-\lambda)Y) \leq \lambda\rho(X) + (1-\lambda)\rho(Y), \quad \forall X, Y \in \mathcal{L}, \lambda \in [0,1]. \tag{4}$$

Convex risk measures are therefore not necessarily either positively homogeneous or subadditive. Although subadditivity and lack of positive homogeneity are both reasonable properties for a risk measure, recall that they are incompatible in most non-trivial situations: an investor must establish which facet of riskiness should be prevailing in his framework. This is an instance of the kind of difficulties one meets in selecting a specific risk measure.

We shall later reconsider the measures described here.

Risk Measures as Imprecise Previsions

When having to assess $\rho(X)$, a subject might identify it with the infimum of the amounts that he would ask to shoulder X. Clearly, the riskier X the higher $\rho(X)$ should be. Since getting a specific amount for receiving X is the same as selling $-X$ for the same amount, $\rho(X)$ can be equivalently viewed as an infimum selling price for $-X$.

Note that in this interpretation, suggested in [11], the subject's action of buying or selling whatever amount is considered in the abstract, in order to better elicit his beliefs. It is therefore not relevant at this stage whether the subject materially has the possibility of concluding the buying/selling operations he imagines.

Regarding $\rho(X)$ as an infimum selling price for $-X$ is equivalent to adopting the behavioural interpretation given in [17] for the upper prevision $\overline{P}$ of $-X$, or also for the opposite of the lower prevision $\underline{P}$ for X, given the conjugacy relation

$$\overline{P}(-X) = -\underline{P}(X) \tag{5}$$

between upper and lower previsions [17]. Therefore we have

$$\rho(X) = \overline{P}(-X) = -\underline{P}(X) \tag{6}$$

From (6), results from the theory of imprecise probabilities[1] can be applied to risk measures.

[1]The term imprecise probabilities includes in its current usage also those concepts, like imprecise (upper or lower) previsions, which are technically more general than imprecise probabilities.

Before passing to this, note that when considering a conditional random variable $X|B$ (B: non-impossible event) the correspondence between risk measures and (conditional) imprecise previsions still holds (replace X with $X|B$ in (5) and (6)). The argument goes along as in the unconditional case, recalling that the buying/selling operations are now considered only when B is true.

Consistency of Risk Measures

Having established the connection between risk measures and imprecise previsions summarised in (6), it is natural to reinterpret existing risk measures in the framework of imprecise probability theory.

The first step consists of applying to them the well known consistency notions developed in [17]. So the question is: is a given risk measure coherent, or does it at least avoid sure loss?

These matters were investigated in [11] for the case of coherent risk measures and VaR. Recalling (6) and referring to upper previsions to make comparisons with pre-existing concepts simpler, the following definition was given:

Definition 3 *Given an arbitrary set D of random variables, a mapping ρ from D into $\mathbb{R}$ is a* coherent risk measure *on D iff there exists a coherent upper prevision $\overline{P}$ defined on $D^* = \{-X : X \in D\}$ such that $\rho(X) = \overline{P}(-X)$.*

It was proved in [11] that

Proposition 1 *When D is a linear space of random variables which contains real constants, a mapping ρ from D into $\mathbb{R}$ is a coherent risk measure according to Definition 2 if and only if it is a coherent risk measure by Definition 3.*

We may therefore conclude that coherent risk measures as defined in [2] are actually a special case of coherent imprecise prevision. Note that Definition 3 is more general, since it operates on *any* (non-empty) set D. On the contrary, a risk measure which satisfies all axioms in Definition 2 on a set D which is not a linear space is *not necessarily* coherent.

For instance, when $D = \{X\}$, $\rho(X) > \sup(-X) = -\inf(X)$ satisfies trivially all axioms but is not coherent on D, since it corresponds to $\overline{P}(-X) > \sup(-X)$, an inequality which violates a necessary condition for coherence (internality, cf. [17]). Note that, by the operational interpretation of risk measures, the inequality $\rho(X) > -\inf(X)$ would imply adding to X, to obtain an acceptable risk, more than the maximum loss X may cause.

The consistency properties of VaR are less clear-cut. Examples may be found where VaR_α is coherent (by Definition 3), but there are other examples where VaR_α does not even avoid sure loss [11] (A risk measure *avoids sure loss* on D if and only if there is an upper prevision $\overline{P}$ that avoids sure loss on $D^* = \{-X : X \in D\}$ such that $\rho(X) = \overline{P}(-X), \forall X \in D$).

It is possible to derive conditions which are either necessary or sufficient for VaR to avoid sure loss or to be coherent. We report one such condition [11]:

Proposition 2 *Let $D = \{X_i\}_{i \in I}$ be a family of arbitrary (bounded) random variables, $\mathbb{P}$ a partition whose atoms describe all values for X_i, $i \in I$, which are jointly possible and P a probability distribution on $\mathbb{P}$. Assign $\alpha \in (0, 1)$.*

If, for some $\omega \in \mathbb{P}$, condition

$$0 < \alpha < \inf_{i \in I} P(X_i \leq X_i(\omega)) \tag{7}$$

holds, then VaR_α avoids sure loss on D.

It can be seen that Proposition 2 holds in several practical circumstances, whilst it is considerably more difficult to comply with sufficient conditions for coherence. An important aspect of all these conditions is that they tend to be verified when α is sufficiently low. However, this is operationally of little use, because:

- α is generally fixed a priori, having the meaning of an assigned confidence level;
- when lowering α, VaR_α tends to increase. If α approaches 0 from above, VaR_α gets close to $-\inf(X)$ from below. But $-\inf(X)$ is too extreme a value for a risk capital (it would certainly cover all losses possibly arising from X, but requiring too much reserve money).

The overall evaluation of VaR as an imprecise prevision is therefore that it is generally not sufficiently dependable.

The case of convex risk measures is rather interesting. In fact, they are seemingly close to coherent risk measures, hence one might wonder whether some corresponding consistency notion could be found in the theory of imprecise previsions, following closely the framework in [17].

The question was tackled in [12] (with further developments in [13]), where a notion of consistency for lower previsions, called *convexity*, was given as follows:

Definition 4 *A mapping* $\underline{P}$ *from* D *into* $\mathbb{R}$ *is a* convex lower prevision *on* D *iff, for all* $n \in \mathbb{N}^+$*, for each* $X_0, X_1, \ldots, X_n \in D$*, for each* $s_1, \ldots, s_n$ *real and non-negative such that* $\sum_{i=1}^n s_i = 1$ (convexity condition)*, defining* $\underline{G} = \sum_{i=1}^n s_i(X_i - \underline{P}(X_i)) - (X_0 - \underline{P}(X_0))$*,* $\sup \underline{G} \geq 0$*.*

Formally, Definition 4 differs from the definition of coherent lower prevision only because of the additional convexity condition $\sum_{i=1}^n s_i = 1$. Therefore every coherent lower prevision is also convex, whilst a convex lower prevision avoids sure loss on D (when $0 \in D$) if and only if $\underline{P}(0) \leq 0$.

Further, it can be shown that (whenever the relevant random variables are in D)

(a) If $\underline{P}(0) \geq 0$, $\underline{P}(\lambda X) \geq \lambda \underline{P}(X)$, $\forall \lambda \in [0, 1]$ and $\underline{P}(\lambda X) \leq \lambda \underline{P}(X)$, $\forall \lambda > 1$

whilst the above inequalities do not necessarily hold when $\underline{P}(0) < 0$ ([13], Sec. 3.1).

Noting that it seems unreasonable in most cases to assign $\underline{P}(0) \neq 0$, a special subclass of convex previsions is singled out:

Definition 5 *A lower prevision* $\underline{P}$ *on* D $(0 \in D)$ *is a* centered convex prevision *(*C-convex prevision*, in short) iff it is convex and* $\underline{P}(0) = 0$*.*

C-convex lower previsions have several nice properties, and are formally a special class of previsions which avoid sure loss but are not necessarily coherent. They are not necessarily positively homogeneous, and (a) above holds for them (but not, in general, for any convex prevision, as already recalled).

Their introduction gives some answers to a broader problem than that of looking for correspondences with convex risk measures. Precisely it tackles the question of detecting, among previsions that avoid sure loss, subclasses with relevant theoretical and operational properties.

Let us return to the relationship with convex risk measures. The following definition was proposed in [12]:

Definition 6 *A mapping* ρ *from* D *into* $\mathbb{R}$ *is a* convex risk measure *on* D *iff for all* $n \in \mathbb{N}^+$*, for each* $X_0, X_1, \ldots, X_n \in D$*, for each* $s_1, \ldots, s_n$ *real and non-negative such that* $\sum_{i=1}^n s_i = 1$*, defining* $\overline{G} = \sum_{i=1}^n s_i(X_i + \rho(X_i)) - (X_0 + \rho(X_0))$*,* $\sup \overline{G} \geq 0$*. Further,* ρ *is a* centered convex risk measure *on* D *iff* ρ *is convex and* $\rho(0) = 0$*.*

Recalling (6) and Definition 4, it is easy to realize that Definition 6 is precisely the definition of convexity for lower previsions, applied to $\rho(X) = -\underline{P}(X)$.

The following characterization theorem holds for convex lower previsions [12]:

Theorem 1 *Let* $\mathcal{L}$ *be a linear space of bounded random variables containing real constants. A mapping* $\underline{P}$ *from* $\mathcal{L}$ *into* $\mathbb{R}$ *is a convex lower prevision on* $\mathcal{L}$ *iff it satisfies the following axioms:*

(T) $\underline{P}(X + c) = \underline{P}(X) + c, \forall X \in \mathcal{L}, \forall c \in \mathbb{R}$ *(translation invariance)*

(M) $\forall X, Y \in \mathcal{L}$*, if* $Y \leq X$ *then* $\underline{P}(Y) \leq \underline{P}(X)$ *(monotonicity)*

(C) $\underline{P}(\lambda X + (1 - \lambda)Y) \geq \lambda \underline{P}(X) + (1 - \lambda)\underline{P}(Y), \forall X, Y \in \mathcal{L}, \forall \lambda \in [0, 1]$ *(concavity).*

It is not difficult then to realize that:

Theorem 2 *If* D *is a linear space of bounded random variables containing real constants, a mapping* ρ *from* D *into* $\mathbb{R}$ *is a convex risk measure (according to Definition 6) iff it satisfies the definition of convex risk measure given by Föllmer and Schied in [5, 6].*

Hence we obtain the traditional definition of convex risk measure as a special case of Definition 6. Again, when D is arbitrary verifying the axioms (for convex risk measures on linear spaces) does not indeed guarantee convexity in the sense of Definition 6. Further, a convex risk measure in the sense of [5] is *not necessarily* centered, a feature which might be hard to justify in many practical situations. Therefore, the notion of C-convex risk measure in Definition 6 seems appropriate to express lack of positive homogeneity on arbitrary sets of random variables.

A subsequent research direction concerns the possibility of generalizing convexity and C-convexity to the conditional case, i.e. when the set D is formed by conditional random variables of the kind $X|B$, where the set of conditioning events is again arbitrary, with the only restriction that $B \neq \phi$ (while some B may be equal to Ω, that is, unconditional random variables may be included into D as well).

This problem was investigated in [13, 14], where conditional convex previsions were introduced and their properties were studied. Again, one of the applications of conditional convex previsions is that of supplying a tool for evaluating conditional risks, i.e. the riskiness of any $X|B$ in a given family D. Note that this kind of problem may arise naturally in a number of situations, including the case of an unconditional risk evaluation made by means of convex risk measures on which some conditioning is performed at a later stage.

The definition of conditional convex lower prevision, generalising Definition 4, is:

Definition 7 *Let D be a set of conditional random variables. $\underline{P} : D \to \mathbb{R}$ is a* convex conditional lower prevision *on D iff, for all $n \in \mathbb{N}^+$, $\forall X_0|B_0, \ldots, X_n|B_n \in \mathcal{D}$, $\forall s_1, \ldots, s_n$ real and non-negative such that $\sum_{i=1}^{n} s_i = 1$, defining $\underline{G} = \sum_{i=1}^{n} s_i B_i(X_i - \underline{P}(X_i|B_i)) - B_0(X_0 - \underline{P}(X_0|B_0))$ and $S(\underline{s}) = \bigvee\{B_i : s_i \neq 0, i = 1, \ldots, n\}$, $\sup\{\underline{G}|S(\underline{s}) \vee B_0\} \geq 0$.*

When omitting the convexity condition in this definition, we obtain, in an equivalent version, the definition of coherent lower prevision due to Williams [20].

It is also possible to derive a generalisation of Theorem 1:

Theorem 3 *Let $\mathcal{X}$ be a linear space of bounded random variables, $\mathcal{E} \subset \mathcal{X}$ the set of all indicator functions of events in $\mathcal{X}$. Let also $1 \in \mathcal{E}$ and $BX \in \mathcal{X}$, $\forall B \in \mathcal{E}$, $\forall X \in \mathcal{X}$.*[2] *Define $\mathcal{E}^{\varnothing} = \mathcal{E} - \{\varnothing\}$, $\mathcal{D}_{LIN} = \{X|B : X \in \mathcal{X}, B \in \mathcal{E}^{\varnothing}\}$. $\underline{P} : \mathcal{D}_{LIN} \to \mathbb{R}$ is a convex conditional lower prevision if and only if:*

(D1) $\underline{P}(X|B) - \underline{P}(Y|B) \leq \sup\{X - Y|B\}, \forall X, Y \in \mathcal{X}, \forall B \in \mathcal{E}^{\varnothing}$

(D2) $\underline{P}(\lambda X + (1 - \lambda)Y|B) \geq \lambda\underline{P}(X|B) + (1 - \lambda)\underline{P}(Y|B), \forall X, Y \in \mathcal{X}, \forall B \in \mathcal{E}^{\varnothing}, \lambda \in]0, 1[$

(D3) $\underline{P}(A(X - \underline{P}(X|A \wedge B))|B) = 0, \forall X \in \mathcal{X}, \forall A, B \in \mathcal{E}^{\varnothing}: A \wedge B \neq \varnothing$.

Condition $(D3)$ in this theorem is especially interesting, since it is (a general version of) what was called in [17] the *Generalised Bayes Rule* (GBR). The GBR therefore *holds also outside coherence*.

As for $(D1)$, it implies axiom (T) in Theorem 1, or also, in the language of risk measures, the translation invariance axiom in Definition 2. This axiom can however be justified autonomously, showing that it is necessary to allow the operational meaning of risk measures discussed before.

Also in the conditional case, the class of centered previsions has more satisfactory consistency requirements (this is not patent from Theorem 3, cf. [13] for details). The generalisation of the centering condition which proves to be sound in the conditional environment is:

[2]The assumptions imply that if A and $B \in \mathcal{E}$ then $A \wedge B$ and $A \vee B \in \mathcal{E}$.

Definition 8 *$\underline{P} : D \to \mathbb{R}$ is a* centered *(conditional) lower prevision if, $\forall X|B \in D$, $0|B \in D$ and $\underline{P}(0|B) = 0$.*

Fundamental Notions of Imprecise Probability Theory in Risk Measurement

In this section we discuss the application and extension of some basic notions from imprecise probability theory to risk measurement.

First we consider the concept of *natural extension* which, as well known, allows extending any coherent imprecise prevision on D onto any superset $D' \supset D$ and correcting any prevision that avoids sure loss into a coherent one, the correction being *least-committal*, i.e. the natural extension $\underline{E}$ of a lower prevision $\underline{P}$ is such that $\underline{E}(X) \geq \underline{P}(X), \forall X \in D$ and that every coherent $\underline{P}^* \geq \underline{P}$ is such that $\underline{P}^* \geq \underline{E}$.

The natural extension can be applied in risk measurement:

- to extend a coherent risk measure on any superset
- to correct a risk measure which is not coherent but avoids sure loss into a coherent risk measure.

However, there might be some practical constraints which prevent us from applying the natural extension. In fact, using (6) and $\underline{E}(X) \geq \underline{P}(X)$ we get $\underline{E}(X) \leq \rho(X), \forall X \in D$ (ρ *dominates* $\underline{E}$). This means that the natural extension is *less prudential* than ρ, since it requires allocating a smaller amount of money than ρ to cover the same risks (its being least-committal guarantees at any rate that it is the most prudential among the coherent corrections of ρ which are less prudential than ρ itself). Using the natural extension might be questioned by some authorities (or *regulators*) who would rather prefer a correction more prudential than ρ. A solution to this problem is to perform the correction in the opposite direction, i.e. to find some *upper extension* $\underline{U}$ such that: $\underline{U}(X) \geq \rho(X)$, $\underline{U}$ is coherent and is in some sense an optimal correction among the coherent risk measures that dominate ρ. It is shown in [11] that this problem can be solved, under mild restrictions, by resorting to some concepts developed in [19] and extending a result proved there.

Another point is that the natural extension cannot be applied to correct previsions which do not avoid sure loss, since it is infinite in this case. With respect to this, a similar concept arising from the theory of convex previsions, the *convex natural extension*, can be helpful.

Definition 9 *Let* $\underline{P} : D \to \mathbb{R}$ *be a lower prevision,* Z *an arbitrary bounded random variable. Define* $g_i = s_i(X_i - \underline{P}(X_i))$, $L(Z) = \{\alpha : Z - \alpha \geq \sum_{i=1}^n g_i$, *for some* $n \geq 1, X_i \in \mathcal{D}, s_i \geq 0$, *with* $\sum_{i=1}^n s_i = 1\}$.

$\underline{E}_c(Z) = \sup L(Z)$ *is termed* convex natural extension *of* $\underline{P}$ *on* Z.

The definition differs formally from that of the natural extension in [17] only because of the additional convexity constraint $\sum_{i=1}^n s_i = 1$, and the properties are similar. For instance, a lower prevision is convex (coherent) on D iff it coincides there with its convex natural extension (with its natural extension); hence the convex natural extension characterises convexity, in the same way as the natural extension characterises coherence.

With respect to the correction problem, it can be proved that

Proposition 3 $\underline{E}_c(Z)$ *is finite,* $\forall Z$, *iff* $\underline{P}$ *avoids unbounded sure loss*

where the condition of avoiding unbounded sure loss is defined as follows:

Definition 10 $\underline{P} : \mathcal{D} \to \mathbb{R}$ *is a lower prevision that* avoids unbounded sure loss *on* D *iff there exists* $k \in \mathbb{R}$ *such that, for all* $n \in \mathbb{N}^+$, $\forall X_1, \dots, X_n \in D$, $\forall s_1, \dots, s_n \geq 0$ *with* $\sum_{i=1}^n s_i = 1$, $\sup \sum_{i=1}^n s_i(X_i - \underline{P}(X_i)) \geq k$.

This condition is quite unsatisfactory as a rationality requirement, but is rather mild and considerably larger than avoiding sure loss (for instance, it always holds when D is finite). Therefore it allows using the convex natural extension for performing corrections in cases where the natural extension would not be applicable.

The concept of convex natural extension was generalised to C-convex conditional previsions in [13].

The *envelope theorem* is another important issue in the theory of imprecise previsions [17]. It says that a lower prevision $\underline{P}$ is coherent on D if and only if there exists a (non-empty) set $\mathcal{P}$ of coherent precise previsions[3] such that

$$\underline{P}(X) = \inf_{P \in \mathcal{P}} \{P(X)\}, \forall X \in D \tag{8}$$

(inf is attained). For the version with upper previsions, replace $\underline{P}, \inf$ with $\overline{P}, \sup$.

The envelope theorem relates the indirect approach to imprecise previsions, defining them in terms of other uncertainty measures (infima or suprema of sets of precise previsions, or of probabilities when all random variables are indicator functions of events), to the direct one, which gives a definition corresponding to a direct behavioural interpretation in certain betting schemes. Another very important feature of the envelope theorem is that it points out a way of assessing imprecise previsions, which is often applied in practice. Instances of envelope-like theorems appear also in many other different areas, like for instance convex analysis [15].

In risk measurement, an envelope theorem is mentioned, for instance, in [2, 3]. This theorem is a special case of the envelope theorem recalled above, because D is a linear space $\mathcal{L}$, and $\mathcal{P}$ is a set of expectations, each derived from a precise σ-additive probability. Each expectation on $\mathcal{L}$ is called *scenario*. Observe (cf. the definition recalled in the previous Footnote) that assessing a precise prevision on D does not imply assessing also a precise probability (to be used to compute an expectation which coincides with the prevision), nor must the probability be σ-additive. When each scenario is assessed by an expert, for instance, it is unnecessary to oblige the expert to assign preliminarily a precise probability.

Summing up, a broader envelope theorem for coherent risk measures, corresponding to that in [17], may be stated as follows:

Proposition 4 ρ *is a coherent risk measure on* D *if and only if*

$$\rho(X) = \sup\{P(-X) : P \in \mathcal{P}\} \tag{9}$$

where $\mathcal{P}$ $(\neq \emptyset)$ *is a set of coherent precise previsions on* $D^* = \{-X : X \in D\}$.

A general, interesting question is: what about the envelope theorem when coherence is replaced by convexity? The answer is given by the following[4]

Theorem 4 $\underline{P}$ *is convex on* D *iff there exist a set* $\mathcal{P}$ *of coherent precise previsions on* D *and a function* $\alpha : \mathcal{P} \to \mathbb{R}$ *such that:*

(a) $\underline{P}(X) = \inf_{P \in \mathcal{P}} \{P(X) + \alpha(P)\}$, $\forall X \in D$ *(*$\inf$ *is attained).*

[3]The notion of coherent *precise* prevision given by de Finetti, cf. [4], says that P is a coherent precise prevision on D iff $\forall n \in \mathbb{N}^+, \forall X_1, \dots, X_n \in D, \forall s_1, \dots, s_n \in \mathbb{R}$, defining $G = \sum_{i=1}^n s_i(X_i - P(X_i))$, $\sup G \geq 0$.

[4]We report the theorem as stated in [12]. Versions of this theorem appeared also in [5].

Moreover, $\underline{P}$ is C-convex iff ($0 \in D$ and) both (a) and the following (b) hold:

(b) $\inf_{P\in\mathcal{P}} \alpha(P) = 0$ ($\inf$ is attained).

The customary envelope theorem for coherent lower previsions is a special case of Theorem 4, with $\alpha \equiv 0$. When the envelope theorem is used for deriving a subject's assessment from evaluations by a group of experts, each assessing a precise prevision, function α may be interpreted as a correction the subject applies to each expert's opinion.

It is also possible to derive envelope theorems for conditional lower previsions [14]. The matter is more complicated, especially because conditioning events are allowed to (possibly) have zero probability. The simplest theorem, working when no zero probabilities are involved, is the following one:

Theorem 5 *Let $\mathcal{B} = \{B : \exists X|B \in D\}$, $\mathcal{P}$ be a set of coherent precise previsions on $D \cup \mathcal{B}$ such that $\forall P \in \mathcal{P}$, $P(B) > 0\ \forall B \in \mathcal{B}$, and let $\alpha : \mathcal{P} \to \mathbb{R}$ be a real function. Then*

$$\underline{P}(X|B) = \inf_{P\in\mathcal{P}} \{P(X|B) + \frac{\alpha(P)}{P(B)}\}\ \forall X|B \in \mathcal{D} \quad (10)$$

is a convex conditional lower prevision on D, whenever the infimum in (10) is finite. Further, $\underline{P}$ is centered iff $\inf_{P\in\mathcal{P}}\{\frac{\alpha(P)}{P(B)}\} = 0,\ \forall B \in \mathcal{B}$.

Other envelope theorems are stated in [14].

There is another issue to mention for its relevance in risk measurement, and this is *dilation*, studied in [16]. Suppose that X is conditioned on each of the (non-impossible) events of a given partition $\mathbb{P}$. Roughly speaking, dilation occurs when the uncertainty evaluation on $X|B$ is vaguer then the evaluation on X, *whatever* is $B \in \mathbb{P}^{\varnothing} = \mathbb{P} - \{\varnothing\}$. The case when both lower and upper previsions are assessed is particularly meaningful, since then there is *strict dilation* [16] when

$$\underline{P}(X|B) < \underline{P}(X) \le \overline{P}(X) < \overline{P}(X|B),\ \forall B \in \mathbb{P} \quad (11)$$

and we say that $\mathbb{P}$ *dilates strictly* X, while $\mathbb{P}$ *dilates* X when one of the strict inequalities in (11) may be replaced by a weak inequality. However assuming as usual $\overline{P}(X|B) = -\underline{P}(-X|B)$, which specialises to (5) when $B = \Omega$, strict dilation can be discussed also referring to lower or alternatively upper previsions only. This is the case of risk measures, where (11) is written as follows

$$\rho(X|B) > \rho(X), \rho(-X|B) > \rho(-X), \forall B \in \mathbb{P}^{\varnothing}. \quad (12)$$

In words, strict dilation implies that the money an investor should reserve to cover risks from his holding either X or $-X$ must be increased when assuming that B will be true, no matter which $B \in \mathbb{P}^{\varnothing}$ is chosen. Since one $B \in \mathbb{P}^{\varnothing}$ is certainly true, the reserve money should be raised in all cases. This is disturbing, and a possible way-out is to observe that the argument is conditioned on the given partition. Hence changing the partition does not imply that dilation will still occur, and a well-chosen partition might let us avoid dilation in certain problems. Dilation was shown in [16] to be a relatively common phenomenon with coherent imprecise probabilities, and results in [16] can be extended to convex previsions (and hence to convex risk measures, cf. [14]). This is one of the topics in applying imprecise previsions to risk measurement which require further investigation, but cannot anyway be neglected when considering risk measures for conditional risks.

A different, little investigated question concerns techniques for operationally checking consistency and possibly correcting risk measures for simple random variables. Among various possibilities, a promising approach is that of adapting ideas developed in [18], but further work is needed in this area.

A Note on References

The literature on financial risk measurement is quite large[5]. A good all-purpose (or nearly) reference is currently the web site *www.gloriamundi.org*, whose secondary title is *All about Value-at-Risk*. Although the promise is untenable, this site is actually an excellent starting point for getting material on many other risk measurement topics. It can be used, for instance, to trace papers on special cases of coherent risk measures or anyway on other risk measures not mentioned here, like the *Expected Shortfall* or *Conditional Value-at-Risk*. Essential ideas on the (traditional) approach to coherent risk measures may be found in [1, 2, 3]. References [7, 8] are among the papers discussing further aspects of coherent risk measures, motivations for adopting them, and related topics. Convex risk measures were introduced and studied in [5, 6]. There is much less literature on the relationship between risk measures and imprecise probabilities. This paper is

[5]This section aims at giving some guidance for a first approach to the theme of this paper. As such it does not mention certain more advanced or particular aspects, nor the related references.

based on [11, 12, 13, 14], where proofs, details and additional material are available. The basic concepts about imprecise previsions, assumed known and recalled only in passing here, are those of [17]. For a different unifying approach to risk measures, imprecise probabilities, and other concepts, see [9]. In a sense, convex risk measures are *bounded rationality* models (when viewed from coherence); for other alternatives to coherence, see [10], [17], Appendix B, and, for a discussion, [13], Sec. 3.4.

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A Report on ISIPTA '05

The Fourth International Symposium on Imprecise Probabilities and Their Applications (ISIPTA '05) was held during July 20-23 2005, in Pittsburth, Pennsylvania, United States. The meeting followed the format of the previous ISIPTAs, and fostered active discussion on several topics related to imprecision in probability values. A considerable amount of information about the symposium can be found at the site www.sipta.org/isipta05, including online proceedings with all papers presented at the symposium. Here we just note a few highlights.

Three tutorials were presented in the afternoon of July 20. The first tutorial, by Gert de Cooman, offered an introduction to imprecise probabilities. The second tutorial, by Paolo Vicig, presented connections between imprecise probabilities and finance — we note that the contents of the tutorial appear on this very issue of the SIPTA Newsletter. The third tutorial, by Kurt Weichselberger, discussed the presenter's theory of interval probability — a paper based on the contents of the tutorial appears in the proceedings.

The format of the symposium aimed at fostering communication among participants. There were no parallel sessions, and every paper was first presented orally and later as a poster. Every author had the chance to give a short account of results and contributions; specific questions and detailed discussions were then deferred to the poster session. A total of 42 papers were presented in various plenary sessions, covering themes from elicitation to computation.

Two distinguished speakers gave invited talks. Isaac Levi discussed the requirement of convexity often imposed on sets of probability measures, arguing that this requirement should be adopted by all practitioners. Arthur Dempster addressed the difficult question of how to model ignorance, particulary from the perspective of Dempster-Shafer theory.

Participants at ISIPTA '05 were invited to participate, at no cost, in a Workshop on Financial Risk Assessment held during the morning of July 24. Four invited talks were given on risk measures and risk analysis. The abstract of all talks given at the symposium and at the associated workshop are available at the web site www.sipta.org/isipta05.

The symposium took place at the campus of Carnegie Mellon University; the conference banquet was held at the Andy Warhol Museum, one of the main touristic attractions of Pittsburgh. Finally, participants had a nice time on the evening of July 22, when they went on a cruise on the riverboat Keystone Belle.

As required by the articles of SIPTA, a general meeting of the members took place during ISIPTA '05. The following report on the meeting has been produced by Marco Zaffalon, SIPTA's secretary:

The general meeting of SIPTA took place on July 23rd, just at the end of ISIPTA '05. The Members of SIPTA decided on several issues, of which the most important ones are reported below.

- The Members decided to change the name of the Society into "Society for Imprecise Probability: Theories and Applications". In order to have a uniform way to refer both to the Society and to the ISIPTA meetings, the Members decided also to re-name the latter as "International Symposium on Imprecise Probability: Theories and Applications". Of course, the acronyms, SIPTA and ISIPTA, remain the same.

- New SIPTA articles have been approved that substantially differ with respect to the previous articles.

  - Probably the major change consists in the creation of a more open form of Society membership: the new articles establish that at a given point in time, each person who has attended either an ISIPTA conference or a SIPTA school in the previous 4 years, is eligible to become a Member of the Society. This enables quite a large number of people to become now SIPTA Members, and so to be actively involved in the management of the Society. Actually, the transition to the new state of the Society is still in progress for organizational reasons. This means that only in a few months from now the Society will be ready to set up new elections for the executive committee by a voting procedure extended to all the (also new) Members.

  - Another important issue concerns the SIPTA Schools on Imprecise Probabilities. The first such school was held in Lugano (Switzerland) in 2004, and it was motivated by the attempt to spread a wide and deep view of imprecise probability at a relatively accessible level. Yet, at that time the school was not "officially" provided in the articles. In contrast, the new SIPTA articles now deal explicitly with such schools. In particular, the articles establish that a SIPTA school will have to be held every even year, between June and September (and in a location to determine each time). In other words, SIPTA schools and ISIPTAs will alternate in the years to come. It may be important at this point to recall that the next SIPTA school will be held in Madrid on July 24–28, 2006 (as reported in this issue of the newsletter).

- The Members decided also on the location of ISIPTA '07. Among the proposed locations for the next conference, the Members accepted Jirina Vejnarova's proposal to make ISIPTA '07 in Prague. Consequently, Jirina will be the leading figure behind the organization of the next conference.

Announcement: Second SIPTA School on Imprecise Probabilities

Contributed by Enrique Miranda, organizer of the school.

SIPTA is announcing a School on Imprecise Probabilities, aiming at graduate students, post-docs, and faculty without previous knowledge of imprecise probability.

The school is intended as a wide and deep introduction to imprecise probability topics, both theoretical and applied. Specifically, the school will focus on coherent lower previsions and their behavioural interpretation, non-additive measures and applications to decision theory, the imprecise Dirichlet model, predictive inference with imprecise probabilities, and knowledge discovery from data sets under weak assumptions. It will be an aim of the school to connect the mentioned topics into an overall picture within the framework of imprecise probabilities.

Dates and Location

The Second SIPTA School on Imprecise Probabilities will take place in the Headquarters of the Rey Juan Carlos University Foundation, in Madrid (Spain), on July 24-28, 2006. The event is organized by SIPTA and by the Group of Statistics and Decision Sciences (GECD) from Rey Juan Carlos University.

You can find all the relevant information on http://bayes.escet.urjc.es/˜emiranda/sipta.

Schedule and topics

Each of the five days of the school will be devoted to a different topic, and the time will be divided equally between theory and exercises/applications. The topics covered will be:

- The Imprecise Dirichlet Model (Jean-Marc Bernard, Universit Paris V).
- Predictive inference with imprecise probabilities (Gert de Cooman, Ghent University).
- Non-additive measures and applications on decision theory (Jean-Yves Jaffray, Universit Paris VI).
- Coherent lower previsions and their behavioural interpretation (Enrique Miranda, Rey Juan Carlos University).
- Knowledge discovery from data sets under weak assumptions: the case of prior ignorance and incomplete data (Marco Zaffalon, IDSIA).

Registration

If you would like to attend the school on Imprecise Probabilities, please fill in the pre-registration form on http://bayes.escet.urjc.es/˜emiranda/sipta no later than March 31, 2006, enclosing a short CV (no longer than 2 pages). Notification on acceptance will be made on April 15, 2006.

Contact

If you have any questions or remarks, please contact Enrique Miranda by e-mail, at enrique.miranda@urjc.es

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Announcement: Third Int. Conf. on Soft Methods in Probability and Statistics

Contributed by Jonathan Lawry, general chair of the conference.

Over the last thirty years there has been a growing interest in extending the theory of probability and statistics to allow for more flexible modelling of uncertainty, ignorance and fuzziness. Most such extensions result in a "softening" of the classical theory, to allow for imprecision in probability judgements and to incorporate fuzzy constraints and events. Many approaches utilise concepts, tools and techniques developed in theories such as fuzzy set theory, possibility theory, imprecise probability theory and Dempster-Shafer theory.

The need for soft extensions of probability theory is becoming apparent in a wide range of applications areas. For example, in data analysis and data mining it is becoming increasingly clear that integrating fuzzy sets and probability can lead to more robust and interpretable models that better capture both the inherent uncertainty and fuzziness of the underlying data. Also, in science and engineering the need to analyse and model the true uncertainty associated with complex systems requires a more sophisticated representation of ignorance than that provided by uninformative Bayesian priors.

SMPS 2006 aims to provide a forum for researchers to present and discuss ideas, theories, and applications. The scope of conference is to bring together experts representing all existing and novel approaches to soft probability and statistics. In particular, we would welcome papers combining probability and statistics with fuzzy logic, applications of the Dempster-Shafer theory, possibility theory, generalized theories of uncertainty, generalized random elements, generalized probabilities and so on.

Dates and Location

Soft Methods in Probability and Statistics (SMPS) 2006 will be hosted by the Artificial Intelligence Group, Department of Engineering Mathematics at the University of Bristol, UK. The conference will take place in September 5-7 2006. This is the third of a series of biennial conferences organized in 2002 by the Systems Research Institute from the Polish Academy of Sciences in Warsaw and in 2004 by the Department of Statistics and Operation Research at the University of Oviedo in Spain.

Contact

For further information see the conference web page www.enm.bris.ac.uk/SMPS or email smps-2006@bris.ac.uk

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Announcement: IJAR Special Issue on Random Sets and Imprecise Probabilities

Announced at the SIPTA mailing list by Vladik Kreinovich.

Random sets, or measurable multi-valued mappings, constitute a powerful generalisation of random variables with applications on areas as diverse as economics, data mining or environmental sciences. On the other hand, imprecise probability models have arisen in the last years as a powerful alternative to classical probability for situations when imprecision or vagueness prevents the use of a precise and unique probability distribution with certain guarantees.

This special issue is meant to cover the several connections existing between random sets and imprecise probabilities. Foundational issues on this topic include, but are not limited to, upper and lower probabilities (capacities) induced by random sets, sets of measurable selections, upper and lower integrals of random sets and coarse data analysis. Applications of random sets and imprecise probabilities in Computer Science, Economics, Social Sciences or Information Technology, are also welcome.

Authors should submit their papers electronically to both special issue editors. The papers must contain original unpublished work that is not being submitted for publication elsewhere. Due to limitations of space, only papers with less than 20 pages will be considered. Manuscripts submitted to this special issue should follow the format guidelines that can be found at http://www.elsevier.com/locate/ijar, and will be refereed according to the standard procedures for International Journal of Approximate Reasoning.

Deadline and contact

The deadline for submissions is May 15, 2006. The issue is expected to be ready by early 2007. Electronic submission is required. Please e-mail a postscript or pdf file of your manuscript to both special issue editors:

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Announcement: IJAR Special Issue on The Imprecise Dirichlet Model

Announced at the SIPTA mailing list by Jean-Marc Bernard, editor of the special issue.

Since it was first proposed by Walley (1996, JRSS B), the imprecise Dirichlet model (IDM) has attracted a great deal of attention in researchers working in the field of imprecise probabilities, which has already been reflected in several papers published in past ISIPTA proceedings, as well as in Statistics or Artificial Intelligence journals.

The IDM aims at making inferences from multinomial data from a state of prior ignorance. The IDM generalizes the common multinomial/Dirichlet Bayesian model, while answering several difficulties met with Bayesian precise models especially when modeling prior ignorance is required. The IDM can be applied to various problems of inference (parametric or predictive) arising with various types of categorical data (from the most simple ones to, e.g., multiway contingency tables, tree-structured data) and also with numerical data in a nonparametric setting. The aims of this special issue as a whole should be to further investigate the theoretical properties of the IDM, as well as its applications to these problems. Comparisons with other models or other methods of inference are also welcome.

Instructions to authors

The papers should contain original unpublished work not submitted for publication in another journal. Due to limitations of space, the recommended length for papers is 15 pages, and in any case papers should not exceed 20 pages. Submitted manuscripts should follow the format guidelines for IJAR (see http://www.elsevier.com/locate/ijar, especially the "Guide for authors" page). Manuscripts will be refereed according to the standard procedures for IJAR.

Authors should submit their papers electronically by e-mail (as a .ps or a .pdf file) to the special issue editor.

We also encourage potential authors to contact the special issue editor by e-mail as soon as possible to indicate their intention to contribute to the special issue (possibly with the subject or the tentative title of their future contribution).

Deadline

The deadline for submissions is June 30, 2006 The issue is expected to be ready around February-March 2007.

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Software section: CkC – The Check Coherence package

The package CkC (short for "Check-Coherence") offers facilities to check the coherency (consistency) and to perform extensions (inferences) of probabilistic assessments — in fact, partial conditional lower/upper probability assessments.

The system is distributed for noncommercial use at www.dipmat.unipg.it/˜upkd/paid/software.html, where the user will also find documentation and instructions concerning installation. The package runs under Microsoft Windows (32-bit) platforms, and uses the lp_solve package that is freely distributed at http://groups.yahoo.com/group/lp_solve. The package has been coded by M. Baioletti, A. Capotorti, L. Galli, F. Rossi, S. Tognoloni and B. Vantaggi. The manual has been written by A. Capotorti and B. Vantaggi.

The package CkC consists of two pieces: a graphical user interface and an inference engine that actually checks coherency and performs extensions. The user can input probabilistic *and* logical assessments through the specification of *events*, *relations* and *probabilities* in the graphical user interface. There are buttons to add, delete, clear and edit the various pieces of data handled by the package — it is also possible to load assessments from files and to save sessions on files.

An *event* is a description of a conditional event of the domain. A *relation* is a logical constraint among the events. Relations can be nested, including partitions, identities, incompatibility between events, inclusion, and logical connectives of negation, disjunction and conjunction. A *probability* associates conditional events with lower/upper probability values.

When all events, relations and probabilities are ready in the graphical user interface, the package can verify whether the assessments are coherent. The concept of coherency is a technical one, explained and explored at length by Coletti and Scozzafava in a recent book [2]. Roughly speaking, assessments are coherent when it is possible to build a probability measure that dominates them — or, perhaps more intuitively, when the assessments avoid sure loss in a gambling interpretation. Thus if some assessments are coherent, it is possible to compute the upper and lower probabilities for any additional conditional event; that is, it is possible to extend the assessments. CkC can compute extensions, showing results in the graphical user interface. Several examples, extracted from technical papers in the literature, come with the package.

All in all, CkC is easy to use, and very powerful in its unique combination of probability and logic. It is very painful to check coherency manually, as it usually requires considerable symbol manipulation and expertise with linear programming. CkC reduces the whole process to very simple and intuitive steps.

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Software section: The rcdd package

The manipulation of intervals and sets of probabilities often leads one to deal with objects such as convex hulls and separating hyperplanes. There are a few packages that handle these quantities for generic computational geometry problems; these packages usually require that problems be translated to their own input languages, and often run standalone, producing output in their own output languages.

The package rcdd, by Geyer, Lazar and Meeden, offers a much needed path between the popular statistical system R and the rather efficient computational geometry package cdd. The web site www.stat.umn.edu/geyer/rcdd describes briefly the package and gives pointers for downloads that are essential to its use.

In short, rcdd is a bridge between R and cddlib. The system R is freely available at www.r-project.org/; this system is widely used as it offers a wide set of statistical tools, a powerful set of graphical capabilities, and a programming language for mathematical calculations. The cdd package and the associated library cddlib have been coded by K. Fukuda and are distributed at www.ifor.math.ethz.ch/~fukuda/cdd_home/. These pieces of software aim at producing the list of vertices and rays of a polyhedron given by a system of linear inequalities, and at computing the convex hull of points; thus one can convert back and forth from inequalities to points. The package and library deal with degenerate cases and use multiple precision arithmetic; thus they are a very robust piece of code.

The package rcdd hides all complexity of cddlib inside a set of functions in the R language. The user simply installs rcdd and calls the relevant functionality using standard data structures in R. Dealing with probabilistic assessmens of lower/upper probabilities and expectations is much facilitated by the clean syntax of the R language. The graphical resources of the R system can then be used directly on the output of cddlib.

The authors of rcdd have published a paper at ISIPTA '05 describing the functionality of rcdd [1]; the paper is by far the best way to get acquainted with the package and to see examples of its use.

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