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Message from the editor

This issue of the SIPTA newsletter opens with a fascinating "History Section": a contribution by Prof. Terrence Fine as he looks at the thirtieth anniversary of his influential book *Theories of Probability: An Examination of Foundations*, published in 1973. Prof. Fine was kind enough to prepare a written essay on this topic to the Newsletter.

We also have a generous contribution by Matthias Troffaes, in which he reviews various criteria for decision making with imprecise probabilities. His gentle discussion of the field is a most welcome piece to everyone interested in imprecise probabilities.

This issue of the newsletter brings fresh information on the coming Fourth International Symposium on Imprecise Probabilities and their Applications (ISIPTA'05), to occur in Pittsburgh, Pennsylvania, United States.

A new section is added to the Newsletter in this issue, containing abstracts of recently published papers on imprecise probabilities.

Finally, we have a very interesting Software Section on the *TBMLAB*, a package for manipulation of belief functions in the Transferable Belief Model. We must thank Prof. Smets, one of the authors of this package, for contributing in record speed to the Newsletter.

If you have contributions to make to this Newsletter, or if you know of any event or publication that should be of interest to members of SIPTA, please let me know (send a message to fgcozman@usp.br).

Cheers!

Fabio G. Cozman

History section: Reflections of the Thirtieth Anniversary of *Theories of Probability: An Examination of Foundations (TOP)*

By Terrence L. Fine
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The ISIPTA'03 banquet at the beautiful Castelgrande in Bellinzona, coming exactly thirty years after I gave a copy of the then just-published TOP to Andrei Kolmogorov at a USSR information theory conference in Talinn, presented an opportunity for reflection on how I now see TOP [3]. What follows is more of such a reflection than was the banquet talk.

Openness to a Broad Range of Interpretations of Probability

TOP was unusual in its taking seriously a broad range of views of probability. While it identified flaws in all of the interpretations suggested for probability, it did not see these as grounds for their rejection. Arguing correctly, as many have, that a particular probability interpretation they dislike has serious flaws does not then support the merits of the interpretation they prefer. We have not been given a finite list of mutually exclusive interpretations, one and

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only one of which is correct. Almost all other authors at the time felt compelled to eventually defend a single interpretation of probability. The position of TOP was that all of the different interpretations were independently useful, all of them were flawed, and a choice depended upon the type of phenomena you were looking at.

Detailed Study of What it Means to Commit to Standard Numerical Probability

TOP was also unusual in its inquiries into the bases for and possible mathematical formulations of these concepts of probability. It did not take numerical probability for granted as a representation of probability. Effort was devoted in Chapters 2 and 3 to finding a standpoint from which it could be seen what assumptions were required to justify the usual numerical probability. No other book was as insistent as TOP on the need to ground the mathematical model in the meaning of the concept and to be open to mathematical models that were not the familiar one. Here I was guided by the work on the theory of measurement scales as developed by Patrick Suppes [7] and mathematical psychologists. An "empirical" domain of physical events or of (rational or individual) beliefs supporting some notion of "probable" or "at least as probable as" is to be mapped into a mathematical domain (say of events) together with an assignment of events (or pairs of events) into the reals with the arithmetic properties being meaningful.

In particular, Chapter 2 was devoted to a study of comparative probability in which events A and B are ordered as to their likelihood, "A is as least as likely to occur as is B" or "$A \succeq B$". A careful study of the question whether, given reasonable axioms on the binary relation $\succeq$, there then must exist a numerical probability measure P that agrees with the ordering in that $A \succeq B \iff P(A) \geq P(B)$ reveals that this is not the case even in the simple case of a sample space with as few as five outcomes.

Conditions that ensure the existence of a representation by P are quite restrictive. Halpern [5] counterexamples my Theorem II.8 where I attempted to use Cox-like arguments to provide conditions under which there would exist a conditional probability representation for the conditional version of "A given B is as least as likely as C given D". Halpern shows that Cox-like arguments are not easily made to work to assert the existence of representing probability measures, particularly when the sample space is finite. While pointing out an error in TOP, Halpern's results strengthen our basic argument by showing the restrictiveness of conditions that are needed to ensure representations by the familiar probability measures.

Upper and Lower Probability

When I wrote TOP I cited ideas of Keynes, Koopman, Smith, and Good, but provided no discussion of the mathematical model of upper and lower probability. A curious omission given that several monographs appeared not long after that focussed on just this model! Kyburg published [8] in 1974 and gave a mathematically detailed argument for the use of upper and lower probability for epistemic probability that is closely coupled to statistical relative frequencies. This ground-breaking monograph was less influential than it should have been due to its forbidding mathematical logic style. Shafer published the engaging and highly influential [12] in 1976 introducing belief (lower probability) and plausibility (upper probability) functions for reasoning with beliefs. Shafer's monograph motivated me to embark on a long series of investigations with my students into upper and lower probability focussed instead on their use in modeling frequentist phenomena. This work culminated in [11], although easier access to the ideas is provided in [4]. The most detailed development of upper and lower probability, but extended to upper and lower expectations or previsions, is that of Walley in his masterpiece [13] that has been so influential in research by those associated with SIPTA.

Focus on Physical Probability Related to Relative Frequencies of Occurrence

Given my background as an electrical engineer, I was particularly interested in relative frequency based concepts of probability that were prevalent in the physical sciences (e.g., Chapter 4). While theories of belief and of rational argument are clearly important, so is the need for a reconstruction of probabilistic reasoning in engineering and the physical sciences. Unfortunately, this is an area that has been ceded to standard probability with little objection. My research on upper and lower probability ([11, 4]) examined the convergence or divergence of time averages of stationary bounded random variables in an eventually successful attempt to achieve statements other than the unique one of convergence with probability one imposed by the Stationarity Convergence Theorem. Recently I returned to the study of frequentist models by using the characterization of previsions in terms of a set of measures. My student Fierens and I attempt to explain a set of measures in frequentist terms (see [2] and [1]) in which it is the full set of measures that is the correct description and not any individual measure in the set.

Progress

Major progress on the topics discussed in TOP has occurred in several areas. The discussion of computational complexity, randomness, and probability in Chapter 5 was perhaps the first such in a monograph. A huge amount of research since then is beautifully presented by Li and Vitanyi [9] and completely replaces what we had to say in Chapter 5.

There has also been major progress on subjective probability, the subject of Chapter 8. This has come in the form of much discussion of beliefs represented by classical probability as well as major improvements in our abilities to model complex problems through such devices as Bayesian belief propagation networks introduced by Judea Pearl ([10] is his most recent monograph) and in our ability to effectively compute Bayes solutions. One can view Jaynes' work in the framework of our Chapter 6 as well as Chapter 8. Jaynes' posthumous [6] provides a comprehensive exposition of his (often controversial) views. Of particular interest to SIPTA has been the great development of upper and lower previsions (expectations) by Walley [13] as providing a far more realistic and well-founded approach to the mathematics of representing beliefs and making rational decisions.

Looked at after thirty years, I still find many of the remaining discussions in TOP illuminating.

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Decision Making with Imprecise Probabilities: A Short Review

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Abstract

Through a simple numerical example, various (but certainly not all) ways to do decision making with imprecise probabilities, and the differences between these ways, are demonstrated.

Introduction

Often, we find ourselves in a situation where we have to make some decision d, which we may freely choose from a finite set D of available decisions. Usually, we do not choose d arbitrarily in D: indeed, we wish to make a decision that performs best according to some criterion, *i.e.*, an *optimal* decision. It is commonly assumed that each decision d induces a real-valued gain J_d: in that case, a decision d is considered optimal in D if it induces the highest gain among all decisions in D. So, we wish to identify the set $\mathrm{opt}\,(D)$ of all decisions that induce the highest gain.

If there is no uncertainty regarding the gains J_d, $d \in D$, then the solution is simply

$$\mathrm{opt}\,(D) = \arg\max_{d \in D} J_d. \tag{1}$$

However, in many situations, the gains J_d induced by decisions d in D are influenced by variables whose values are uncertain. Assuming that these variables can be modelled through a random variable X that takes values in a finite set $\mathcal{X}$ (the *possibility space*), it is customary to consider the gain J_d as a so-called *gamble* on X, that is, we view J_d as a real-valued gain that is a function of X, and that is expressed in a fixed utility scale. So, J_d is an $\mathcal{X}$–$\mathbb{R}$-mapping, interpreted as an uncertain gain: taking decision d, we receive an amount of utility $J_d(x)$ when x turns out to be the realisation of X. For the sake of simplicity, we shall assume that the outcome x of X is independent of the decision d we take: this is called *act-state independence*. What decision should we take?

Irrespective of our beliefs about X, a decision d in D is not optimal if its gain gamble J_d is *point-wise dominated* by a gain gamble J_e for some e in D, *i.e.*, if $J_e \geq J_d$ and $J_e \neq J_d$ for some e in D: choosing e guarantees a higher gain than choosing d, regardless of the realisation of X. So, as a first selection, let us remove all decisions from D whose gain gambles are point-wise dominated (see Berger [1, Section 1.3.2, Definition 5 ff., p. 10]):

$$\begin{aligned}&\mathrm{opt}_{\geq}\,(D) := \\ &\quad \{d \in D \colon (\forall e \in D)(J_e \not\geq J_d \text{ or } J_e = J_d)\}\end{aligned} \tag{2}$$

We recover Eq. (1) if there is no uncertainty regarding the gains J_d, *i.e.*, if all J_d are constant functions of X.

We shall try to answer the following question: given additional information about X, how can we further reduce $\mathrm{opt}_{\geq}\,(D)$?

Maximising Expected Utility?

In the literature, beliefs about X are often modelled by a probability mass function p on $\mathcal{X}$, *i.e.*, a $[0,1]$-valued mapping p on $\mathcal{X}$ whose values sum to one. In that case, a common way to arrive at a set of optimal decisions, goes through the *expected utility* of the gain gambles J_d:

$$\mathbf{E}_p(J_d) := \sum_{x \in \mathcal{X}} p(x) J_d(x)$$

Recall that we have assumed act-state independence: p is independent of d. As far as it makes sense to rank decisions according to the expected utility of their gain gambles, we should *maximise expected utility* (see for instance Raiffa and Schlaifer [7, Section 1.1.4, p. 6], Levi [6, Section 4.8, p. 96, ll. 23–26], or Berger [1, Section 1.5.2, Paragraph I, p. 17]):

$$\mathrm{opt}_{\mathbf{E}_p}\,(D) := \arg\max_{d \in \mathrm{opt}_{\geq}(D)} \mathbf{E}_p(J_a). \tag{3}$$

Unfortunately, it may happen that our beliefs about X cannot be modelled by a probability mass function p, simply because we have insufficient information to identify the probability $p(x)$ of each possible realisation x of X. In such a situation, maximising expected utility may fail to give an adequate representation of optimality.

Example 1 Let X be the unknown outcome of the tossing of a coin; say we *only* know that the outcome will be either heads or tails (so $\mathcal{X} = \{H, T\}$), and that the probability of heads lays between 28% and 70%. Consider the decision set $D = \{1, 2, 3, 4, 5, 6\}$ and the gain gambles

$$\begin{aligned}
J_1(H) &= 4, & J_1(T) &= 0, \\
J_2(H) &= 0, & J_2(T) &= 4, \\
J_3(H) &= 3, & J_3(T) &= 2, \\
J_4(H) &= \tfrac{1}{2}, & J_4(T) &= 3, \\
J_5(H) &= \tfrac{47}{20}, & J_5(T) &= \tfrac{47}{20}, \\
J_6(H) &= \tfrac{41}{10}, & J_6(T) &= -\tfrac{3}{10},
\end{aligned}$$

Clearly, $\mathrm{opt}_{\geq}(D) = \{1, 2, 3, 4, 5, 6\}$, and

$$\mathrm{opt}_{\mathbf{E}_p}(D) = \begin{cases} \{2\}, & \text{if } p(H) < \frac{2}{5}, \\ \{2,3\}, & \text{if } p(H) = \frac{2}{5}, \\ \{3\}, & \text{if } \frac{2}{5} < p(H) < \frac{2}{3}, \\ \{1,3\}, & \text{if } p(H) = \frac{2}{3}, \\ \{1\}, & \text{if } p(H) > \frac{2}{3}. \end{cases}$$

Concluding, if we have *no* additional information about X, but still insist on using a particular (and necessarily arbitrary) p, which is only required to satisfy $0.28 \leq p(H) \leq 0.7$, we find that $\mathrm{opt}_{\mathbf{E}_p}(D)$ is not very robust against changes in p. This shows that maximising expected utility fails to give an adequate representation of optimality in case of ignorance about the precise value of p.

Generalising to Imprecise Probabilities

Of course, if p can be identified, nothing is wrong with Eq. (3). We shall therefore try to generalise Eq. (3). In doing so, we shall assume that our beliefs about X are modelled by a real-valued mapping $\underline{P}$ defined on a finite set $\mathcal{K}$ of gambles, that represents our assessment of the *lower expected utility* $\underline{P}(f)$ for each gamble f in $\mathcal{K}$;[1] note that $\mathcal{K}$ can be chosen empty if we are completely ignorant. Essentially, this means that instead of a single probability mass function, we now identify a closed convex set $\mathcal{M}$ of probability mass functions, described by the linear inequalities

$$(\forall x \in \mathcal{X})(0 \leq p(x) \leq 1), \sum_{x \in \mathcal{X}} p(x) = 1, \text{ and}$$
$$(\forall f \in \mathcal{K})(\underline{P}(f) \leq \mathbf{E}_p(f)).$$

For a given gamble g, not necessarily in $\mathcal{K}$, we may also derive a lower expected utility $\underline{\mathbf{E}}_{\underline{P}}(g)$ by minimising $\mathbf{E}_p(g)$ subject to the above constraints. This simply amounts to solving a linear program. Perhaps it is worth mentioning that this linear program corresponds exactly to the so-called *natural extension* of the mapping $\underline{P}$, when $\underline{P}(f)$ is interpreted as a supremum buying price for f (see Walley [10, Section 3.4.1, p. 136]). In the literature, $\mathcal{M}$ is called a *credal set* (see for instance Giron and Rios [4], and Levi [6, Section 4.2, pp. 76–78], for more comments on this model), and $\underline{P}$, in the supremum buying price interpretation, is called a *lower prevision* (because they generalise the previsions, which are fair prices, of De Finetti [2, Vol. I, Section 3.1, pp. 69–75]).

[1]The upper expected utility of a gamble f is $\overline{P}(f)$ if and only if the lower expected utility of $-f$ is $-\overline{P}(f)$. So, for any gamble f in $\mathcal{K}$, $\underline{P}(-f) = -\overline{P}(f)$, and therefore, without loss of generality, we can restrict ourselves to lower expected utility.

Although the belief model described above is not very general, it is sufficiently general to model both expected utility and complete ignorance: these two extremes are obtained by taking $\mathcal{M}$ either equal to a singleton, or equal to the set of all probability mass functions (*i.e.*, $\mathcal{K} = \emptyset$). It also allows us to demonstrate the differences between different ways to do decision making with imprecise probabilities on the example we presented before.

In that example, the given information can be modelled by, say, a lower prevision $\underline{P}$ on $\mathcal{K} = \{I_H, -I_H\}$, defined by $\underline{P}(I_H) = 0.28$ and $\underline{P}(-I_H) = -0.7$, where I_H is the gamble defined by $I_H(H) = 1$ and $I_H(T) = 0$. For this $\underline{P}$, the set $\mathcal{M}$ corresponds exactly to the set of all probabilities p such that $0.28 \leq p(H) \leq 0.7$. We also easily find for any gamble f on X that

$$\underline{\mathbf{E}}_{\underline{P}}(f) = \min\{0.28f(H) + 0.72f(T), \\ 0.7f(H) + 0.3f(T)\}.$$

Γ-*Maximin and* Γ-*Maximax*

As a very simple way to generalise Eq. (3), we could take the lower expected utility $\underline{\mathbf{E}}_{\underline{P}}$ as a replacement for the expected utility $\mathbf{E}_p$ (see for instance Gilboa and Schmeidler [3], or Berger [1, Section 4.7.6, pp. 215–223]):

$$\mathrm{opt}_{\underline{\mathbf{E}}_{\underline{P}}}(D) := \arg\max_{d \in \mathrm{opt}_{\geq}(D)} \underline{\mathbf{E}}_{\underline{P}}(J_d); \tag{4}$$

this criterion is called Γ-*maximin*, and amounts to worst-case optimisation: we take a decision that maximises the worst expected gain. Applied on Example 1, we find as a solution $\mathrm{opt}_{\underline{\mathbf{E}}_{\underline{P}}}(D) = \{5\}$.

In case $\mathcal{K} = \emptyset$, *i.e.*, in case of complete ignorance, Γ-maximin coincides with maximin (see Berger [1, Eq. (4.96), p. 216]).

Some authors consider best-case optimisation, taking a decision that maximises the best expected gain (see for instance Satia and Lave [8]). In our example, the "Γ-maximax" solution is $\mathrm{opt}_{\overline{\mathbf{E}}_{\underline{P}}}(D) = \{2\}$.

Maximality and Interval Dominance

Eq. (3) is essentially the result of pair-wise preferences based on expected utility: if we define the strict partial order $>_p$ on D as $d >_p e$ whenever $\mathbf{E}_p(J_d) > \mathbf{E}_p(J_e)$, or equivalently, whenever $\mathbf{E}_p(J_d - J_e) > 0$, then we could simply write

$$\mathrm{opt}_{\mathbf{E}_p}(D) = \max_{>_p}\left(\mathrm{opt}_{\geq}(D)\right),$$

where the operator $\max_{>_p}(\cdot)$ selects the $>_p$-maximal, *i.e.*, the $>_p$-undominated elements from a set with strict partial order $>_p$.

Using the supremum buying price interpretation, it is easy to derive pair-wise preferences from $\underline{P}$: define $>_{\underline{P}}$ as $d >_{\underline{P}} e$ whenever $\underline{\mathbf{E}}_{\underline{P}}(J_d - J_e) > 0$. Indeed, $\underline{\mathbf{E}}_{\underline{P}}(J_d - J_e) > 0$ means that we are disposed to pay a strictly positive price in order to take decision d instead of e, which clearly indicates strict preference of d over e (see Walley [10, Sections 3.9.1–3.9.3, pp. 160–162]). Since $>_{\underline{P}}$ is a strict partial order, we arrive at

$$\begin{aligned}&\mathrm{opt}_{>_{\underline{P}}}(D) := \max_{>_{\underline{P}}}\left(\mathrm{opt}_{\geq}(D)\right), \\ &= \{d \in \mathrm{opt}_{\geq}(D) : \\ &\qquad (\forall e \in \mathrm{opt}_{\geq}(D))(\underline{\mathbf{E}}_{\underline{P}}(J_e - J_d) \leq 0)\}\end{aligned} \tag{5}$$

as another generalisation of Eq. (3), called *maximality*. Note that $>_{\underline{P}}$ can also be viewed as a robustification of $>_p$ over p in $\mathcal{M}$. Applied on Example 1, we find $\mathrm{opt}_{>_{\underline{P}}}(D) = \{1, 2, 3, 5\}$ as a solution.

Another robustification of $>_p$ is the strict partial ordering $\sqsupset_{\underline{P}}$ defined by $d \sqsupset_{\underline{P}} e$ whenever $\underline{\mathbf{E}}_{\underline{P}}(J_d) > \overline{\mathbf{E}}_{\underline{P}}(J_e)$. This ordering is called *interval dominance* (see Zaffalon, Wesnes, and Petrini [11, Section 2.3.3, pp. 68–69] for a brief discussion and references). The resulting notion of optimality is weaker than maximality: applied on Example 1, $\mathrm{opt}_{\sqsupset_{\underline{P}}}(D) = \{1, 2, 3, 5, 6\}$, which is strictly larger than $\mathrm{opt}_{>_{\underline{P}}}(D)$.

E-Admissibility

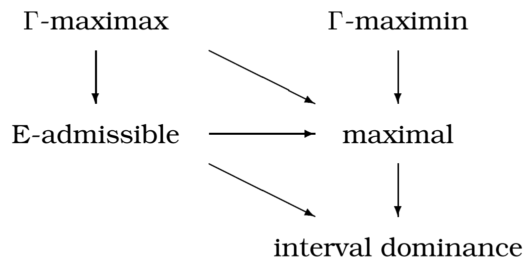
In Example 1, we have shown that $\mathrm{opt}_{\mathbf{E}_p}(D)$ may not be very robust against changes in p. *E-admissibility* (see Good [5, p. 114, ll. 8–9], or Levi [6, Section 4.8, p. 96, ll. 8–20]) robustifies $\mathrm{opt}_{\mathbf{E}_p}(D)$ against changes of p in $\mathcal{M}$:

$$\mathrm{opt}_{\mathcal{M}}(D) := \bigcup_{p \in \mathcal{M}} \mathrm{opt}_{\mathbf{E}_p}(D); \tag{6}$$

this provides a third way to generalise Eq. (3). We find $\mathrm{opt}_{\mathcal{M}}(D) = \{1, 2, 3\}$ for the example.

What Is the Right One?

Apparently, there is no right choice. Instead, what properties do we want our notion of optimality to satisfy? Let us summarise a few important guidelines. First note that, in general, the following implications hold:



as is also demonstrated by Example 1.

E-admissibility, maximality, and interval dominance have the nice property that the more determinate our beliefs (*i.e.*, the smaller $\mathcal{M}$), the smaller the set of optimal actions. In contradistinction, Γ-maximin and Γ-maximax lack this property, and usually only select a single action, even in case of complete ignorance. However, if we are *only* interested in the most pessimistic (or most optimistic) solution, disregarding other reasonable solutions, then Γ-maximin (or Γ-maximax) seems appropriate. Note that Seidenfeld [9] has recently compared Γ-maximin to E-admissibility, and argued against Γ-maximin in sequential decision problems.

If we do not settle for Γ-maximin (or Γ-maximax), should we choose E-admissibility, maximality, or interval dominance? As already mentioned, interval dominance is weaker than maximality, and moreover, it is computationally very similar to maximality. Therefore, we should prefer maximality to interval dominance.

This leaves E-admissibility and maximality. They are quite similar: they coincide on all decision sets D that contain two decisions. In case we consider larger decision sets, they coincide if the set of gain gambles is convex (for instance, if we consider *randomised* decisions). As already mentioned, E-admissibility is stronger than maximality, and also has some other advantages over maximality. For instance, $\frac{1}{5}J_2 + \frac{4}{5}J_3 >_{\underline{P}} J_5$, so, choosing decision 2 with probability 20% and decision 3 with probability 80% is preferred to decision 5. Therefore, we should perhaps not consider decision 5 as optimal. E-admissibility is not vulnerable to such argument.

However, from a computational viewpoint, maximality is apparently to be preferred over E-admissibility, especially if the possibility space is large. In case of n decisions, with gain gambles on a possibility space of size m, maximality requires us to solve at most n^2 linear programs (one for each comparison), each involving

m constraints; the number of variables depends on $\underline{P}$. Even if m gets large, say 10000 or even more, this presents no computational problems, as long as n is not too large. E-admissibility on the other hand, requires us to solve a classical optimisation problem (maximising expected utility) for each p in the m-dimensional polyhedron $\mathcal{M}$: this can be done graphically or analytically in case m is small (*e.g.*, for $m = 2$, as in our example), but in general, it is not clear how to find all solutions as efficiently as with maximality. Unfortunately, the computational aspects of E-admissibility have not received much attention in the literature. Bluntly discretising $\mathcal{M}$ is prone to an exponential growth in sample points with respect to m under fixed precision. Maximality does not suffer from this problem.

In conclusion, the decision criterion to settle for in a particular application, depends at least on the goals of the decision maker (what properties should optimality satisfy?), and possibly also on the size of the problem due to computational issues—although this point certainly requires a more in depth investigation.

Acknowledgements

I especially want to thank Teddy Seidenfeld for the many instructive discussions about maximality versus E-admissibility. I also thank Fabio Cozman for suggestions which improved the quality and the readability of this paper.

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The Fourth International Symposium on Imprecise Probabilities and Their Applications — ISIPTA'05

The Fourth ISIPTA will happen in 2005, following a series started in 1999. Detailed information about the symposium can be found at the site http://www.sipta.org/isipta05/. The remainder of this section presents a few important itens regarding the meeting.

ISIPTA'05 will be held at Carnegie Mellon University, in Pittsburgh, Pennsylvania, United States. Important dates are:

Paper submission deadline: February 25 2005
Notification of acceptance: April 15 2005
Deadline for revised papers: May 15 2005
Symposium: July 20-23 2005

The ISIPTA meetings are one of the primary international forums to present and discuss new results on the theory and applications of imprecise probabilities. The symposium deals with models that include belief functions, Choquet capacities, comparative probability orderings, convex sets of probability measures, fuzzy measures, interval-valued probabilities, possibility measures, plausibility measures, and upper and lower expectations or previsions. Imprecise probability models are needed in inference problems where the relevant information is scarce, vague or conflicting, and in decision problems where preferences may also be incomplete. Applications in psychological, economic and financial models are welcome, as are applications in engineering, management, computer science, statistics, philosophy, and related fields. Although the symposium is open to contributions on all aspects of imprecise probability, three main themes will be emphasised in this edition: decision-making, algorithms, and real applications.

Workshop on Financial Risk Assessment

There will be a workshop addendum to the conference, to be held on July 24, with invited speakers on the topic of financial risk assessment, to which all of the ISIPTA'05 participants are welcome, at no additional registration cost. Details will be announced later in the ISIPTA'05 site.

Submissions and Proceedings

Papers will be accepted only as PDF files with a limit of 8 pages in two-column format. Guidelines are available in a PDF file and in a word file at the site http://www.sipta.org/isipta05/submit.html. The simplest way to produce an article that satisfies these requirements is to use Latex with the isipta2005.sty style, using guidelines at the file isipta05.tex (or using this file as a template). Otherwise, use the file isipta2005.doc as a template.

Submission is electronic; there are directions to input paper data and attach the PDF file through the submission page.

The Program Committee will decide which of the submitted papers are accepted, by carefully evaluating their originality, significance, technical soundness, and clarity of exposition. All the accepted papers will be included in a volume of proceedings, published by Brightdocs.

There will be free electronic access to the proceedings after some time. Before the conference, the electronic access will be restricted to the ISIPTA'05 attendees to allow them to study the papers in some detail before they are actually presented. Each accepted paper will be given the opportunity for both a brief oral presentation as well as a poster session.

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Program committee members are listed in the site www.sipta.org/isipta05.html.

Questions and Secretariat

If you have any questions about the symposium, please contact the Organising Committee preferably by email (teddy@stat.cmu.edu - fgcozman@usp.br), or at the following address:

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Abstracts on Imprecise Probabilities

In this new section, we list published papers that are related to imprecise probabilities. In this first run, we just bring some papers that were announced in the SIPTA mailing list during 2004 — hopefully this is a valid incentive for people to announce their papers in the mailing list! Please start announcing your papers, and let me know about any relevant papers you get to find, by sending a message to fgcozman@usp.br!! *Fabio Cozman*

G. de Cooman, M. Zaffalon. Updating beliefs with incomplete observations. *Artificial Intelligence*, 159(1-2), 75-125, 2004
(at http://www.idsia.ch/~zaffalon/papers/)
Abstract:
Currently, there is renewed interest in the problem, raised by Shafer in 1985, of updating probabilities when observations are incomplete (or set-valued). This is a fundamental problem in general, and of particular interest for Bayesian networks. Recently, Grunwald and Halpern have shown that commonly used updating strategies fail in this case, except under very special assumptions. In this paper we pro-

pose a new method for updating probabilities with incomplete observations. Our approach is deliberately conservative: we make no assumptions about the so-called incompleteness mechanism that associates complete with incomplete observations. We model our ignorance about this mechanism by a vacuous lower prevision, a tool from the theory of imprecise probabilities, and we use only coherence arguments to turn prior into posterior (updated) probabilities. In general, this new approach to updating produces lower and upper posterior probabilities and previsions (expectations), as well as partially determinate decisions. This is a logical consequence of the existing ignorance about the incompleteness mechanism. As an example, we use the new updating method to properly address the apparent paradox in the 'Monty Hall' puzzle. More importantly, we apply it to the problem of classification of new evidence in probabilistic expert systems, where it leads to a new, so-called 'conservative updating rule'. In the special case of Bayesian networks constructed using expert knowledge, we provide an exact algorithm for classification based on our updating rule, which has linear-time complexity for a class of networks wider than polytrees. This result is then extended to the more general framework of credal networks, where computations are often much harder than with Bayesian nets. Using an example, we show that our rule appears to provide a solid basis for reliable updating with incomplete observations, when no strong assumptions about the incompleteness mechanism are justified.

A. Antonucci, A. Salvetti, M. Zaffalon. Assessing debris flow hazard by credal nets. In: M. López-Díaz, M. A. Gil, P. Grzegorzewski, O. Hryniewicz, J. Lawry (editors), *Soft Methodology and Random Information Systems*, Springer, pp. 125-132, 2004.
(at http://www.idsia.ch/~zaffalon/papers/)
Abstract:
Debris flows are destructive natural hazards that affect human life, buildings, and infrastructures. Despite their importance, debris flows are only partially understood, and human expertise still plays a key role for hazard identification. This paper proposes filling the modelling gap by using credal networks, an imprecise-probability model. The model uses a directed graph to capture the causal relationships between the triggering factors of debris flows. Quantitative influences are represented by probability intervals, determined from historical data, expert knowledge, and theoretical models. Most importantly, the model joins the empirical and the quantitative modelling levels, in the direction of more credible inferences. The model is evaluated on real case studies related to dangerous areas of the Ticino Canton, southern Switzerland. The case studies highlight the good capabilities of the model: for all the areas the model produces significant probabilities of hazard.

G. de Cooman, E. Miranda. A weak law of large numbers for coherent lower previsions, *Proc. of the Information Processing and Management of Uncertainty in Knowledge-Based Systems Conference (IPMU)*, pp. 451–458, 2004.
(at http://ippserv.ugent.be/~gert/preprints/wlln.pdf)
Abstract:
We prove a weak law of large numbers for coherent lower previsions. The law is a consequence of the rationality criterion of coherence, and it can be proven under surprisingly weak assumptions. Our treatment also uncovers an interesting connection between the behavioural theory of coherent lower previsions, and Shafer and Vovk's game-theoretic approach to probability theory.

R. Haenni. *Towards a Unifying Theory of Logical and Probabilistic Reasoning.* Tech. Report. Center for Junior Research Fellows, University of Konstanz, 2004.
(at http://heidirolf.0catch.com/haenni.pdf)
Abstract:
Logic and probability theory have both a long history in science. They are mainly rooted in philosophy and mathematics, but are nowadays important tools in many other fields such as computer science and, in particular, artificial intelligence. Some philosophers studied the connection between logical and probabilistic reasoning, and some attempts to combine these disciplines have been made in computer science, but logic and probability theory are still widely considered to be separate theories that are only loosely connected. This paper introduces a new perspective which shows that logical and probabilistic reasoning are not more and not less than two opposite extreme cases of one and the same universal theory of reasoning called probabilistic argumentation.

P. P. Shenoy. *No Double-Counting Semantics for Conditional Independence in Probability Theory*
(http://lark.cc.ku.edu/ pshenoy/Papers/WP307.pdf)
Abstract:
Conditional independence for probability theory has been traditionally interpreted in terms of irrelevance and in terms of factorization of the joint probability distribution. In this paper, I will describe a new semantic for conditional independence in terms of no double counting of uncertain information. There are

several advantages of these new semantics. First, these semantics will provide a new method for building models in domains, such as sensor fusion, where these semantics can be easily applied. Second, the Dempster-Shafer (D-S) theory of belief functions uses the semantics of no-double counting evidence to qualify when it is proper to combine belief functions by Dempsters rule of combination. The semantics of no double counting are paramount in building D-S belief function models. Yet, these semantics are not well understood and remains a mystery for many. Hopefully, the results provided here will explain these semantics and make D-S theory more appealing and practical to practitioners, and facilitate the integration of these two uncertainty calculi.

Software section: TBMLAB: a Demonstrator for the Transferable Belief Model (TBM)

By Philippe Smets
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http://iridia.ulb.ac.be/˜psmets

Introducing TBMLAB

TBMLAB means TBM (for transferable belief model) LABoratory. It is a demonstrator for the TBM, a model for the representation of quantified opinions, based on what Shafer has described in his 1976 book, to which many new concepts, clarifications and results have been added.

This software is a freeware developed mainly with the help of Thierry Denoeux (Université Technologique de Compiègne). It can be downloaded from the next web pages: http://iridia.ulb.ac.be/˜psmets/#G by clicking on the TBMLAB_3.1 entry. How to start TBMLAB is explained in the READ_ME file.

Formally the TBMLAB is a demonstrator for the TBM, but in practice it can be seen as a demonstrator for belief function theory. It covers essentially:

- the probability based belief function theory
  - the theory developed by Dempster and by Shafer
  - the theory of hints of Kohlas and Monney
  - the probabilistic argumentation system of Haenni, Kohlas and Lehmann.
- the non-probabilistic belief function theory
  - the theory presented in Shafer's 1976 book
  - the transferable belief model (TBM) of Smets

It does not cover upper and lower or imprecise probability theory.

Running TBMLAB

TBMLAB is a MATLAB program that runs under MATLAB 5.2 or later versions. It is a self contained program, not a set of MATLAB functions.

Data for TBMLAB can be introduced

- **on-line** through many GUI's or
- in **batch mode** where instructions are loaded from a .txt file.

The user can:

1. use TBMLAB to **compute** belief potentials[2] using some combinations of already existing belief potentials. Results can be stored and saved.
2. ask TBMLAB to **display details** of belief potentials.
3. introduce an **evidential network**, and ask TBMLAB to propagate belief potentials in the network. Two kinds of networks can be created:
   - an evidential network where the belief potentials are defined on product spaces of variables, and
   - a conditional evidential network where the belief potentials are conditional potentials defined on one space given the singletons of a second space.
4. run **tutorials**.
5. run the **TBMLAB Tabulator**, a tabulator specially oriented toward belief function computations.
6. use **specialized modules** centered on particular applications.

[2]A belief potential is any of the functions associated with the belief function, like the basic belief assignment, the plausibility function, the commonality function, the implicability functions, and the canonical weight function.

TBMLAB comes with a User Guide. Many global and contextual helps are available.

TBM tasks under TBMLAB

TBMLAB can perform the next TBM tasks.

1. **VBS.** You select several bba's and several variables. The Valuation Base System algorithm (VBS) of Shafer-Shenoy propagates the beliefs and projects them on the selected variables. Entered beliefs can be both joint beliefs or conditional beliefs.
2. **Graphical Method.** You run the VBS under graphical mode. You get a display of the Evidential Network (EVN) or of the Conditional Evidential Network (CEVN). Beliefs are propagated using the VBS. You can graphically display the pignistic probability function for every variable.
3. **Distinct Combination.** You select a set of bba's. They are combined by a mixture of negations, conjunctions and disjunctions using the negation rule, the conjunctive combination rule or/and the disjunctive combination rule.
4. **Cautious Combination.** You select a set of bba's. They are combined by the cautious conjunctive combination rule which is the commutative, associative and idempotent rule of combination.
5. **General Bayesian Theorem.** The GBT requires a conditional belief on X for every $t_i \in T$. You can enter an a priori belief on T and an a priori belief on X (which corresponds to a doubtful, noisy observation). You can run either the GBT to get the resulting belief on the T space or the DRC to get the resulting belief on the X space (forward or backward propagation). Beliefs on X may be continuous.
6. **Get New Forms.** Given a potential, computed any of {m, b, bel, pl, q, wc, wd, M, Bel, Pl, Q, BetP} functions.
7. **Extension/Marginalization.** Given a potential defined on space X, compute its projection on space Y. It performs both marginalization and vacuous extension.
8. **EvClus.** The EvClus (evidential clustering) algorithm determines the belief potentials that satisfy (as well as possible) constraints encoded in a dissimilarity matrix.
9. **LCP BetP Pl** Given the values of BetP or of pl on the singletons of a variable, TBMLAB determines the q-least committed isopignisitc (BetP case) or isoshadow (pl case) bba.
10. **Best supported set.** Given a bba on a large frame, determine the smallest subset with the largest weighted plausibility. This permits to reduce greatly the set of elements in which the truth belongs.
11. **Demonstrations.** Several demos and tutorials are provided.

Feedback from users is welcome.

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