On the Complexity of Propositional and Relational Credal Networks

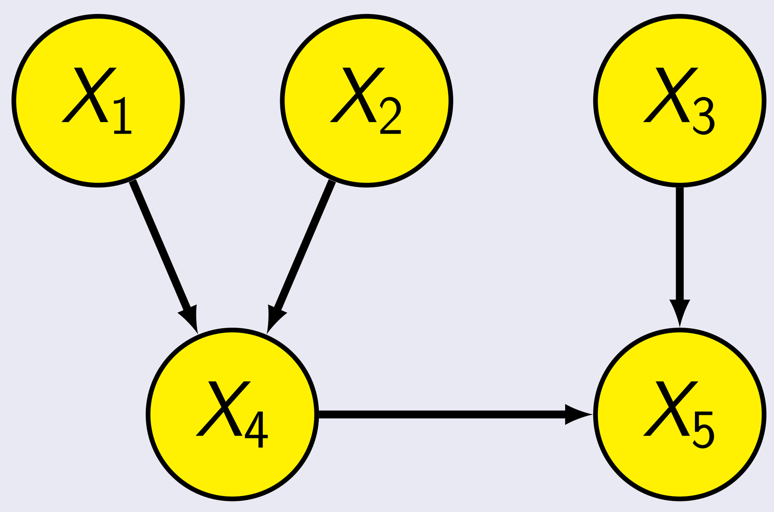
Fabio G. Cozman   Denis D. Mauá
Universidade de São Paulo – Brazil

Goal: to study the relationship between

- specification language and
- complexity in Boolean credal networks.

Credal networks

- Directed acyclic graph, where each node is a random variable with associated "local" credal sets, with associated Markov condition.



- We focus on the *strong extension*:

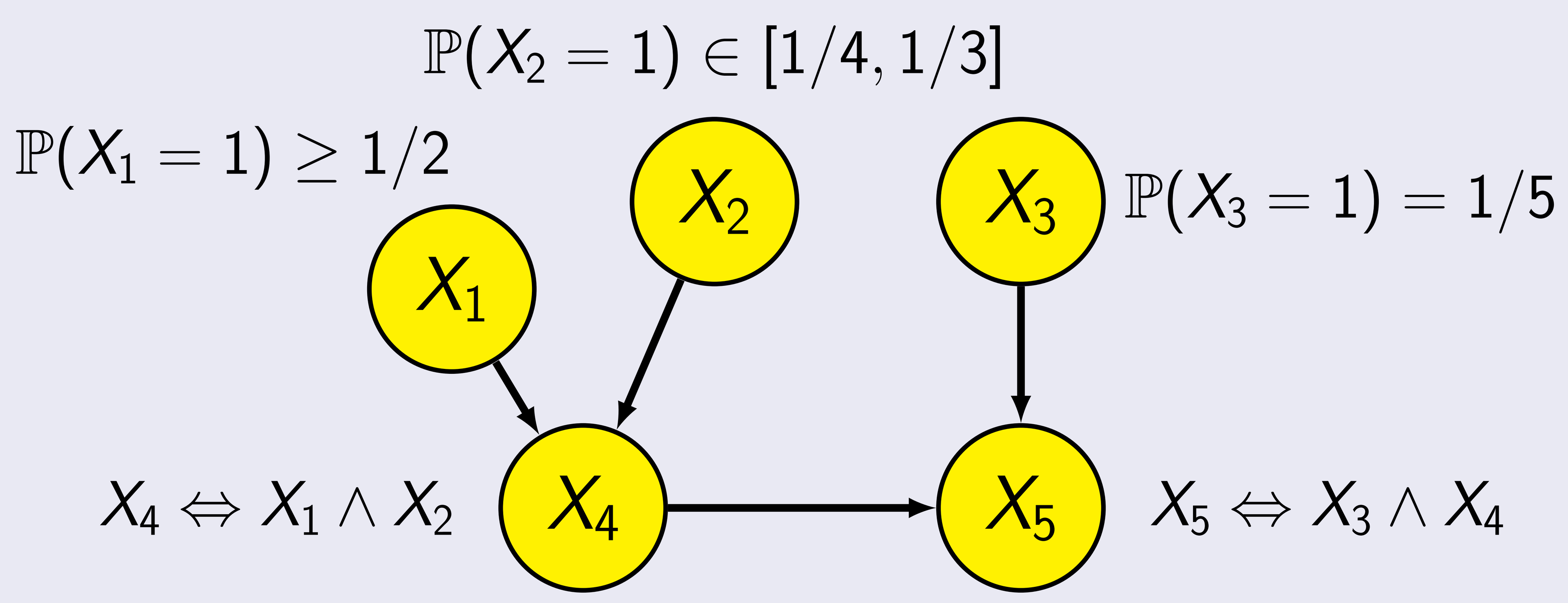
$$\left\{ \mathbb{P} : \mathbb{P}(\mathbf{X} = \mathbf{x}) = \prod_{i=1}^{n} \mathbb{P}(X_i = x_i | \mathrm{pa}(X_i) = \pi_i) \right\}.$$

Complexity

- Marginal inference: $\overline{\mathbb{P}}(\mathbf{X}_Q = \mathbf{x}_Q | \mathbf{X}_E = \mathbf{x}_E) > \gamma$?
- $\mathsf{INF}_d(\mathcal{C})$: the inference problem for a class $\mathcal{C}$ of networks; $\mathsf{INF}_d^+(\mathcal{C})$ when evidence is positive.
  - In Bayesian networks: PP-complete problem.
  - In strong extensions: $\mathsf{NP}^{\mathsf{PP}}$-complete problem.

Specification framework: Propositional

- Associate, with each (Boolean) variable X, either
  - Equivalence $X \Leftrightarrow F(Y_1, \dots, Y_m)$, where F is a sentence in some formal language.
  - Assessment $\mathbb{P}(X = \text{true}) \in [\alpha, \beta]$.



- Every propositional credal network can be specified this way.
- Hence, $\mathsf{INF}_d(\mathsf{Prop}(\wedge, \neg))$ is $\mathsf{NP}^{\mathsf{PP}}$-complete.

Propositional credal networks: Results

Theorem: $\mathsf{INF}_d^+(\mathsf{Prop}(\wedge, (\neg)))$ is polynomial.

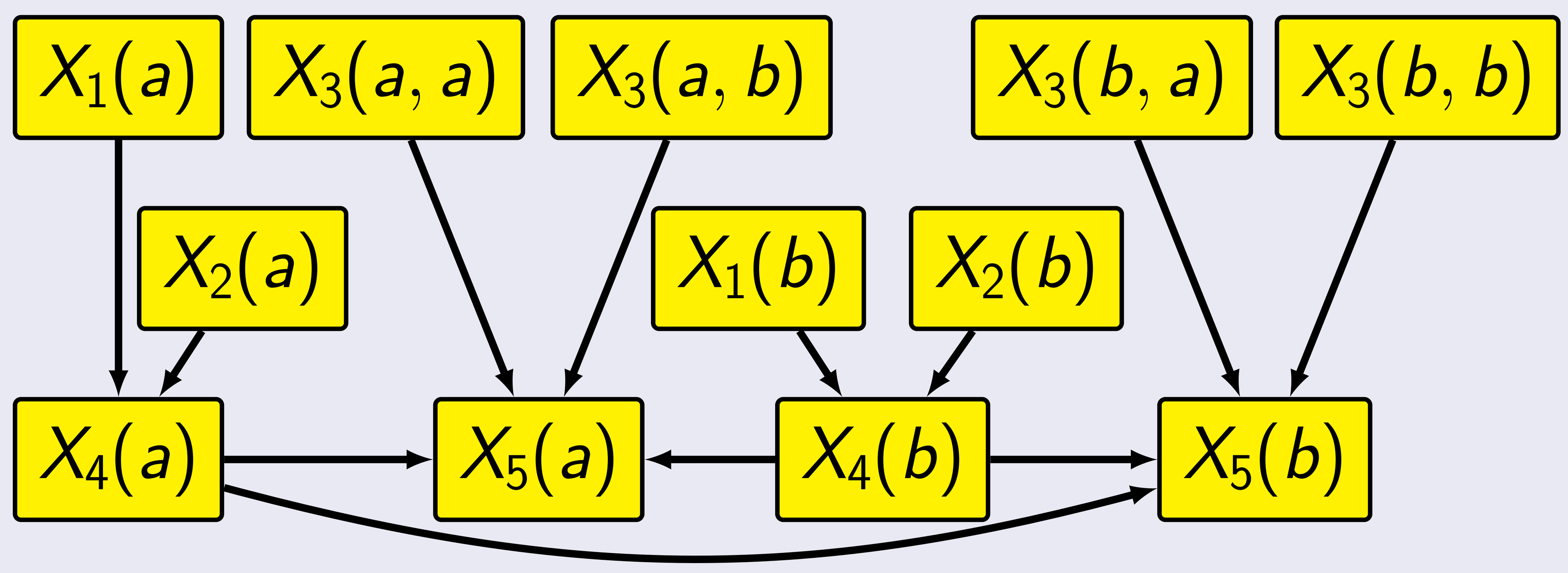
Theorem: $\mathsf{INF}_d^+(\mathsf{Prop}(\wedge, \vee, (\neg)))$ is $\mathsf{NP}^{\mathsf{PP}}$-complete.

Relational credal networks

- Extend: parameterized variables, with logical variables over (finite) domains.
- Example:

$$\mathbb{P}(X_1(x) = 1) \geq 1/2,$$
$$\mathbb{P}(X_2(x) = 1) \in [1/4, 1/3],$$
$$\mathbb{P}(X_3(x, y) = 1) = 1/5,$$
$$X_4(x) \Leftrightarrow X_1(x) \wedge X_2(x),$$
$$X_5(x) \Leftrightarrow \forall y : X_3(x, y) \wedge X_4(y).$$

  with domain $\mathcal{D} = \{a, b\}$:



- Possible semantics:
  - *Coupled parameters*: for each $\gamma \in [1/2, 1]$,
  $$\forall x \in \mathcal{D} : \mathbb{P}(X_1(x)) = \gamma$$
  is a possible assessment.
  - *Decoupled parameters*: for each $x \in \mathcal{D}$,
  $$\mathbb{P}(X_1(x)) = \gamma$$
  is a possible assessment for each $\gamma \in [1/2, 1]$.

Relational credal networks: Results

- Explicit domain is given; inference is $\mathsf{INF}_d(\mathcal{C})$ with respect to grounded network.
- Relations of bounded arity.

Theorem: $\mathsf{INF}_d^+(\mathsf{FFFO})$ is $\mathsf{NP}^{\mathsf{PP}}$-complete both for coupled and decoupled parameters.

- Data complexity DINF_d: inference when model is fixed, and evidence and domain are inputs.

Theorem: $\mathsf{DINF}_d(\mathsf{FFFO})$ is $\mathsf{NP}^{\mathsf{PP}}$-complete for decoupled parameters.

Theorem: $\mathsf{DINF}_d(\mathsf{FFFO})$ is PP-complete for coupled parameters.

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