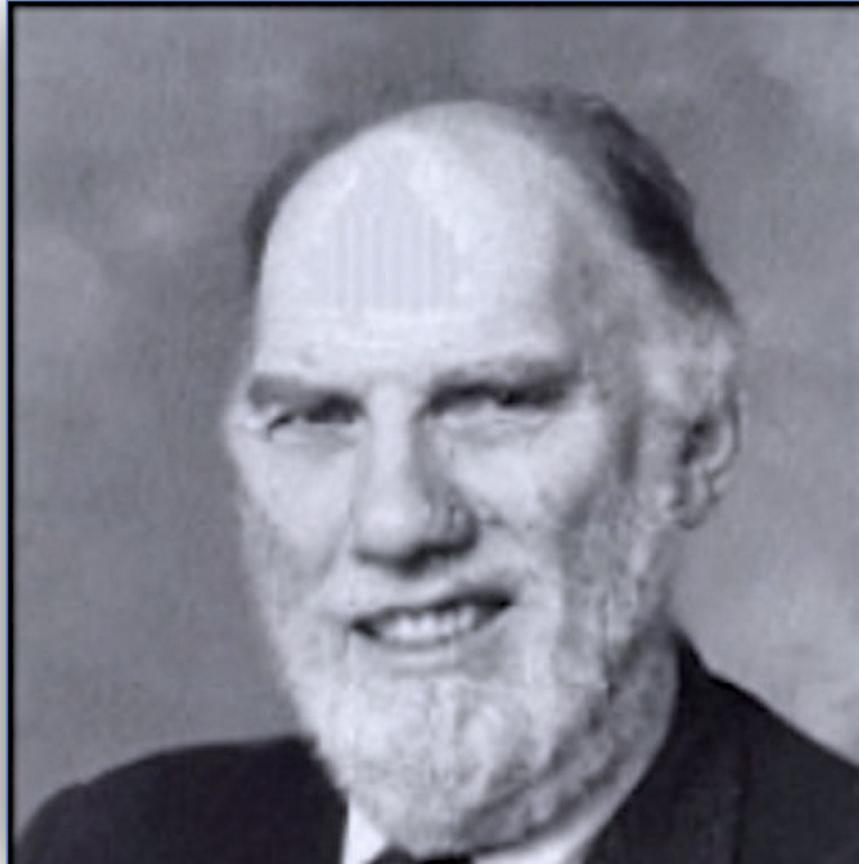


Isaac Levi



1930 – 2018

Outline of this retrospective for ISIPTA-19

Comments about Isaac's 4 ISIPTA papers/presentations

ISIPTA-99 – *Imprecise and Indeterminate probabilities*

ISIPTA-03 – *Extensions of Expected Utility and some limitations of pairwise comparisons (with SSK)*

ISIPTA-05 – *Convexity and E-admissibility in rational choice*

ISIPTA-09 – *Busting Bayes: Learning from Henry Kyburg.*

ISIPTA-99 – *Imprecise and Indeterminate probabilities*

- *SIPTA* uses the wrong *I* and really needs two *I*'s – *SIIPTA*

Isaac begins with a high-level distinction between *commitments* and *performances*.

Commitments are normative ideals.

Example: Full belief, also credence functions ought to respect logical equivalence, under the norm “Be coherent!”

Performances reflect limitations of our real abilities and inabilities,

which *may* interfere with commitments:

Example: Rational agents display full beliefs that are logically inconsistent and so do not have coherent credences.

- See, Section 2.1 of *The Fixation of Belief and Its Undoing* [1991].

A credence function *imprecise* if it is incompletely elicited or only partially identified – an issue of *performance* – which may occur *without* violating the norm to be coherent

Familiar limitations in human abilities make *imprecision* inevitable.

Even with a *determinate* credence function, e.g., with a de Finetti *Prevision* function, a person may specify probability values to some fixed number of decimal places.

Nonetheless, an *imprecisely* identified credence function remains subject to the norms (the *commitments*) for a rational credence function.

- If rounding to 5 decimal places creates de Finetti incoherence, that is also a *normative* failure from perspective of de Finetti's commitments.

- By contrast, an *indeterminate* credence is one that has different norms for decision making and different commitments in decisions compared with canonical (determinate) Bayesian theory.
- Levi's *Indeterminate* credence is represented by a specific (convex) set P of probabilities.

That is not an example of an incomplete elicitation – it's not an *Imprecise* credence.

- From Isaac's perspective, SIPTA's IP theory is mostly about *Indeterminate* NOT *Imprecise* probability.
- Levi's normative decision rule of *E-admissibility* used with *Indeterminate* probabilities, operationalizes the difference between *indeterminate* and *imprecise* probabilities.

Use the distinction between *Imprecision* and *Indeterminacy* to explain de Finetti's well-known opposition to IP theory.

- **Central idea:** De Finetti's *Fundamental Theorem of Prevision* constrains extending a *determinate but imprecisely* defined coherent prevision function P to a *determinate but more fully* defined coherent prevision function P^* .

de Finetti does not abandon his commitment to coherence with his Fundamental Theorem.

Even an imprecisely defined de Finetti prevision does not justify, e.g., the modal choices in the "Ellsberg Urn" decision problem.

ISIPTA-03 – Extensions of Expected Utility and some limitations of pairwise comparisons (with SSK)

This paper contrasts three decision rules that extend determinate EU Theory with a (convex) set \mathcal{P} of probabilities: Γ -Maximin, Maximality, and E-admissibility.

Γ -Maximin (many advocates!): variables ordered by lower (infimum) expectation w.r.t \mathcal{P} .

Maximality (Sen/Walley) is a basic-binary relation where

random variable X is admissible from menu \mathcal{M} provided

$$X \in \mathcal{M} \text{ and there is no } Y \in \mathcal{M} \text{ where } \forall P \in \mathcal{P}, E_P(Y) > E_P(X).$$

E-admissibility (Levi) random variable X is admissible (Bayes) from menu M provided

$$X \in \mathcal{M} \text{ and } \exists P \in \mathcal{P}, \forall Y \in \mathcal{M} \quad E_P(X) \geq E_P(Y)$$

These decision rules have different operational content, as demonstrated in terms of their abilities to distinguish between different convex sets of probabilities.

Even when the menu, the option set, is convex, one decision rule (*E-admissibility*) distinguishes among more convex sets of probabilities than either of the other two.

One important reason why is that *E-admissibility*, alone among these three rules, is not based on pairwise comparisons among options – it's a non-binary *choice function*.

- **Option X may be *E-inadmissible* from menu \mathcal{M} despite the absence of an option Y in \mathcal{M} that is strictly preferred to X in a pairwise choice between X and Y .**
- **One upshot is that *E-admissibility*, but neither of these other two decision rules, may distinguish between pairs of convex sets of probabilities that intersect all the same supporting hyperplanes.**

The ISIPTA-03 paper illustrates two convex sets of probabilities, intersecting all the same supporting hyperplanes, that differ by one extreme but not exposed point.

ISIPTA-05 – *Convexity and E-admissibility in Rational Choice*

ISIPTA-05 took place at CMU (Pittsburgh, PA, USA).

There were five invited speakers: 3 tutorials and 2 “sermons”

T1 Kurt Weichselberger – *The Logical Concept of Probability and Statistical Inference*

T2 Gert de Cooman – *Introduction to Imprecise Probabilities*

T3 Paulo Vicig – *Imprecise Probabilities and Financial Risk Measurement*

S1 Art Dempster gave a plenary – *Probability and the Problem of Ignorance*

S2 Isaac gave an after-dinner talk in the Grand Entrance Hall of the *Andy Warhol Museum*.

Convexity and E-admissibility in Rational Choice

Standing between two enormous Warhol images of *Marilyn Monroe*,



Isaac gave an unqualified defense of his version of E-admissibility where:

***uncertainty* is represented by a *convex* set of (f.a.) probabilities \mathcal{P} ;**

values are represented by a *convex* set of cardinal utilities \mathcal{U} ;

***E-admissibility* – admissible options are *Bayes* – applies with the
cross-product $\mathcal{P} \times \mathcal{U}$.**

Isaac's sermon begins:

Many of the participants in the ISIPTA meetings share in common the sense that they have finally begun to come out of the wilderness to which deviation from the strict Bayesian orthodoxy has banished them. This does not mean, however, that this organization has sought to replace one orthodoxy by another. ISIPTA is one place where many flowers bloom. Even so, many of us, I suspect, wonder whether relaxing the bonds of Bayesian orthodoxy will threaten us with conceptual anarchy. There are so many ways to diverge from Bayesianism that the rebel may feel burdened with an embarrassment of riches. To keep my own activities from lapsing into anarchist chaos, I have sought to follow certain maxims both regarding an account of probabilistic and statistical reasoning and regarding decision-making.

- Maxim 1: Agents ought not to be obliged rationally to endorse credal probability judgments representable by real valued probability functions, evaluations of outcomes of actions representable by real valued functions and evaluations of actions representable by real valued functions.
- Maxim 2: Rational agents ought to remain as faithful to Bayesian ideas of rationally coherent probability judgment and decision making as possible subject to maxim 1.
- Maxim 3: The credal probability judgments made by a rational agent ought to be supported by the information contained in the agent's state of full belief or certainty according to the standard for such support endorsed by the agent.
- Maxim 4: Adopt an Aristotelian rather than a Hegelian view of the logic of belief. There is neither a logic of change of full belief nor a logic of change of probabilistic belief. Legitimate changes in states of belief are either the result of the application of general rules that are subject to revision or changes due to deliberate choice in order to realize specific goals. There are no conditions of diachronic rationality. Only synchronic ones .
- Maxim 5: Within the constraints imposed by the first three maxims, the principles of rational full belief, value judgment and decision making ought to be maximally permissive.

And Isaac ends his sermon,

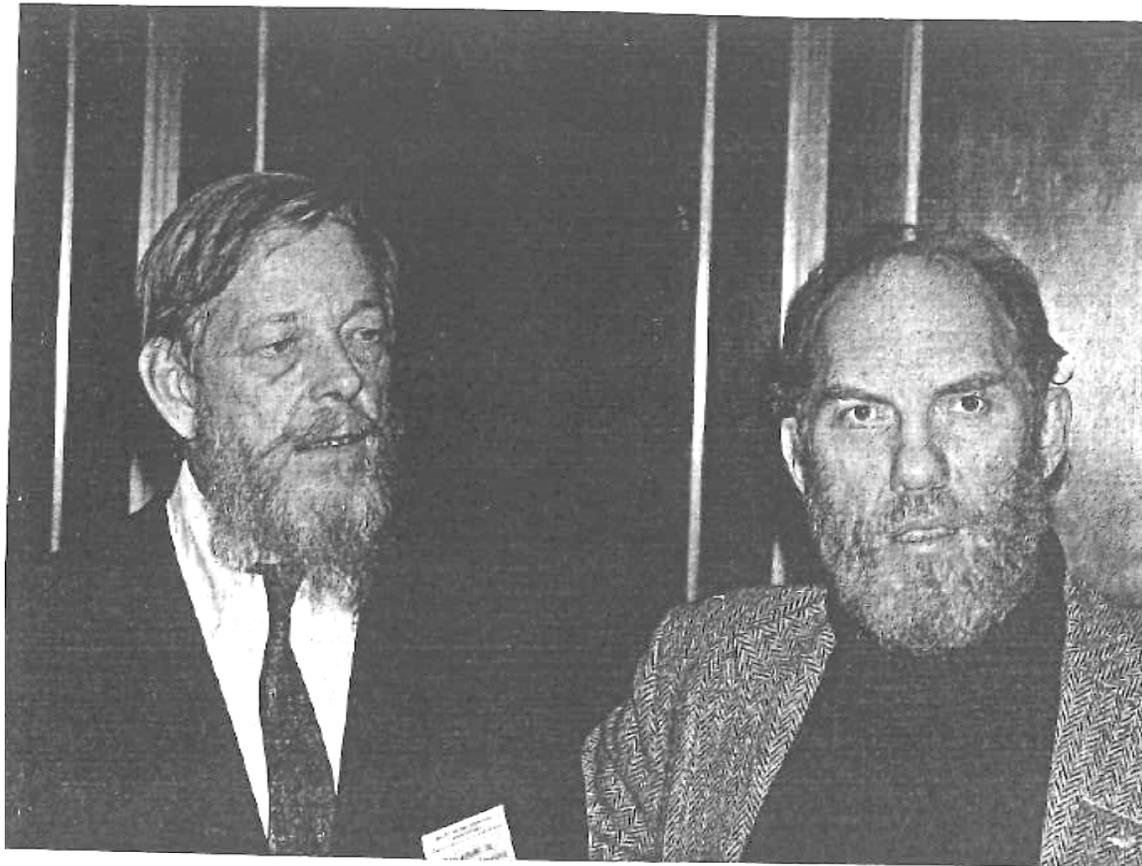
The bottom line is that the convexity of the value structure for the available options is an expression of the idea that the decision maker should avoid ruling out as impermissible any potential resolution of the conflict in his or her values. This recommendation is supported I think by maxim 4. If this is right, the conception of doubt or consensus favored by using the Cross Product Rule is to be recommended over the approach of Seidenfeld, Kadane and Schervish.

The capacity of E-admissibility to discriminate between non-convex and convex sets enveloped by the same upper and lower expectations cannot be used to undermine the requirement of confirmational convexity.

I have been arguing that confirmational convexity is an expression of the view that principles of rational probability and utility judgment should impose only minimal restrictions consonant with qualified Bayesianism. Seidenfeld and Seidenfeld, Kadane and Schervish have appealed to quite similar considerations in order to relax the convexity requirement.

I believe that the pivotal bones of contention concern my insistence that potential resolutions of conflict in the value structure for the options be permissible and the separability of probability and utility for the purpose of deriving the permissible expectation functions in that value structure. Far from being an excessive restriction on rational probability judgment, I submit that confirmational convexity is an expression of the way we should acknowledge our doubts and ignorance as to which probability functions to use in calculating expectations. But I dare say my good friends and colleagues and most profound critics will continue to disagree.

ISIPTA-09 – *Busting Bayes: Learning from Henry Kyburg.*



Left: Henry E. Kyburg, Jr. ; right: Isaac Levi.

In his fourth and final SIPTA presentation, at ISIPTA-09 (Durham), Isaac discharged the bittersweet duty of memorializing his longtime friend and intellectual competitor, Henry Kyburg.

Here I summarize what, to my mind, is a core disagreement between them about the extent to which IP should generalize Bayesian theory.

In the following example (*Direct Inference*, 1977 JoP), Isaac pinpoints how a central principle of Henry's *Epistemological Probability* conflicts with basic Bayesian theory.

Kyburg's theory bases probability on what is known about frequency information in different reference populations.

For example, the frequency information that:

% A among B is in the interval $[l, u]$

fixes *Epistemological Probability* assertions of the form.

***Epist. Prob.* of $A(t)$ is $[l, u]$**

where ' t ' denotes an individual, that is known to be an element of the reference population, B .

The epistemological challenge is how to reconcile competing frequency information about A 's from different reference populations $\{B_1, \dots, B_n\}$, where the investigator knows:

% A among B_j is in the interval $[l_j, u_j]$

and where individual t is known to belong to each population, B_j ($j = 1, \dots, n$).

- What is the *Epist. Prob.* of $A(t)$ w.r.t. this body of frequency information?**

One of Kyburg's rule, the *Strength Rule*, does most of the work in resolving the competing frequency information.

Heuristic Idea for the Strength Rule:

Generally, where one reference set is a proper subset of another, $B_1 \subset B_2$, then that $t \in B_1$ entails $t \in B_2$.

Then, by the *Total Evidence Principle*, priority goes to the frequency information from the narrower reference set B_1 ,

% A among B_1 is in the interval $[l_1, u_1]$

over

% A among B_2 is in the interval $[l_2, u_2]$.

However, this priority is reversed when also $l_1 < l_2$ and $u_2 < u_1$.

Then the broader reference class B_2 carries more informative frequency information about the A's than does the narrower reference set B_1

- The *Strength Rule* gives priority to the broader reference set when that provides more informative frequency information

Isaac pinpoints how the *Strength Rule* conflicts with basic Bayesian theory.

Suppose that an agent's corpus of relevant knowledge about Peterson, K , contains the following three items of information:

- (1) 90% of Swedes are Protestants.
- (2) Either 85%, or 91%, or 95% of Swedish residents of Malmo are Protestants.
- (3) Petersen is a Swedish resident of Malmo.

By the *Strength Rule*, relative to K , *Epist Prob.* "Peterson is a Protestant" is [.90, .90], as the narrower reference set – Swedish residents of Malmo – carries less precise frequency information about being Protestant.

Next, consider these three simple statistical hypotheses, three versions of (2)

- (2.1) 85% of Swedish residents of Malmo are Protestants.
- (2.2) 91% of Swedish residents of Malmo are Protestants.
- (2.3) 95% of Swedish residents of Malmo are Protestants.

Relative to these three consistent expansions of K by simple stat. hypotheses:

$K + (2.1)$ *Epist Prob.* “Peterson is a Protestant” is [.85, .85],

$K + (2.2)$ *Epist Prob.* “Peterson is a Protestant” is [.91, .91],

and $K + (2.3)$ *Epist Prob.* “Peterson is a Protestant” is [.95, .95],

But by the *Strength Rule*, relative to these two consistent expansions of K :

$K + [(2.1) \text{ or } (2.3)]$ *Epist Prob.* “Peterson is a Protestant” is [.90, .90],

$K + [(2.1) \text{ or } (2.2)]$ *Epist Prob.* “Peterson is a Protestant” is [.90, .90],

But K is equivalent to $K + [(2.1) \text{ or } (2.2) \text{ or } (2.3)]$.

Recall that relative to K *Epist Prob.* “Peterson is a Protestant” is [.90, .90],

- There is no Bayes model – no unconditional/conditional probability – that agrees with these 6 *determinate Epistemological Probabilities*.

Isaac used similar thinking to criticize Dempster’s implied theory of *Direct Inference*, and combined these two in his evaluation of Kyburg’s and Dempster’s (different) reconstructions of Fisher’s *Fiducial Probability*.

On Levi's contributions to contemporary Philosophy as a *Pragmatist*.

In the printed version of this paper I discuss Isaac's original treatments of:

Acceptance and Belief Revision – infallibility versus incorrigibility

Social Agents – sidestepping metaphysics using a

common model of rationality for all kinds of agents!

These contributions are enhanced by his use of *indeterminacy*:

indeterminate beliefs and indeterminate values are all part of Epistemology.

In the jargon of 20th century Epistemology, Levi is overtly an

- *Anti-foundationalist*: There are no privileged starting points for acquiring knowledge.

Rather, Epistemology becomes a matter of using one's existing epistemic resources (beliefs and values) for improving one's situation – no matter how those epistemic resources were acquired.

- Such improvements may create new certainties and refined values.

A distinguishing feature of Levi's *Pragmatism* is that epistemic improvements are identified with decision-theoretic tools and, in the spirit of John Dewey, those decision-theoretic tools are part of a unified methodology for improving our situation as humans.

Our wonderful conference organizers have created a “*book*” for Isaac, which will be available at poster sessions, today through Saturday.

- **Please consider writing your name in Isaac’s *book*, and entering a thought to share with Isaac’s immediate family: Judy, Jon, and David.**



Isaac Levi, Teddy Seidenfeld, Henry Kyburg