Imprecise and Indeterminate Probabilities

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Abstract

Bayesian advocates of expected utility maximization use sets of probability distributions to represent very different ideas. *Strict Bayesians* insist that probability judgment is numerically determinate even though the agent can represent such judgments only in imprecise terms. According to *Quasi Bayesians* rational agents may make indeterminate subjective probability judgments. Both kinds of Bayesians require that admissible options maximize expected utility according to some probability distribution. Quasi Bayesians permit the distribution to vary with the context of choice. *Maximalists* allow for choices that do not maximize expected utility against any distribution. *Maximiners* mandate what maximalists allow. This paper defends the quasi Bayesian view against strict Bayesians on the one hand and maximalists and maximiners on the other.

Keywords. Strict Bayesian, Quasi Bayesian, *E*-admissibility, *E*-maximality, Maximizing lower expectation.

1 Introduction

Suppose that decision-maker X judges that his available options belong to set S. Set S is itself a subset of a set $\mathbf{M}(\Omega)$ of probability mixtures of a finite subsets of Ω. X's values, goals and beliefs commit him somehow to an evaluation of the elements of $\mathbf{M}(\Omega)$ as better or worse. This evaluation is representable (by us and not necessarily by X) by a set of weak orderings of $\mathbf{M}(\Omega)$ satisfying the requirements imposed by von Neumann and Morgenstern on the evaluation of lotteries. Each of these weak orderings is representable by a utility function unique up to a positive affine transformation. Consequently we can define the *value structure* $V[\mathbf{M}(\Omega)]$ to be the set

of such *permissible utility functions* or the *permissible von Neumann-Morgenstern preferences* they represent.

For any finite nonempty subset S of $\mathbf{M}(\Omega)$, $V(S)$ is the set of restrictions of the members of $V[\mathbf{M}(\Omega)]$ to the domain S. This is the value structure for S and it consists of permissible utility functions for S. Thus, the value structure $V[\mathbf{M}(\Omega)]$ determines what the value structure $V(S)$ *would be were S* the set of options X judged to be available to him in a given situation.

Let H be a set of propositions such that the decision-maker is sure that exactly one element of H is true. Moreover, if the decision maker X adds any proposition s asserting that some option in S is going to be implemented to what he is certain is true, the result is consistent with each and every element of H.

Let O represent possible outcomes of implementing one or another of the available options in S. The propositions characterizing such outcomes specify information X cares about according to his goals and values. The deductive consequences of X's body of certainties <u>K</u> and the assumption that s is implemented while state h in H is true entails that exactly one consequence in O is true. This is so for each s and each H.

The *extended value structure EV(O)* is representable by a set of utility functions defined for elements of O. Each of these utility functions may be extended to the set $\mathbf{M}(O)$ of all mixtures of elements of O. Each of the permissible utility functions in $EV[\mathbf{M}(O)]$ represents a weak ordering of the members of $\mathbf{M}(O)$ satisfying von Neumann-Morgenstern requirements.

A state of credal probability judgment (credal state) is representable by a set B of permissible probability measures over a given algebra of propositions representing the states of nature in H. This set B

satisfies a convexity condition. (See [6] 78-9 for an explanation of this condition.)

Given the credal state *B* for the states and an extended value structure *EV*(O) for the possible consequences, we can derive a value structure *V*(S) for the options in S by adopting the following two principles:

Cross Product Rule *The set of permissible probability-utility pairs is the set of pairs in B Ä EV(O).*

Expected Utility Rule *Exp*(S) *is the set of permissible expected utility functions defined for elements of S by using the permissible probability-utility pairs obtained from B and V(O) by the cross product rule.*

The Principle of Expected utility: *V*(*S*) = *Exp*(*S*).

To illustrate the use of these abstractly formulated ideas, consider the following example:

Illustrative Decision Problem:

First Version

Suppose that X confronts a choice between three options:

	H_1	H_2
G_1	\$55	-\$45
G_2	-\$45	\$55
R	\$0	\$0

For the sake of the argument, I shall suppose that utility of outcomes is linear in money so that the dollar amounts represent utilities as well as outcomes in $O = Y$. In this discussion, I shall focus on probability judgment and shall therefore take the utilities of outcomes to be precisely and determinately given.

Here $S = \Omega = \{G_1, G_2, R\}$ and $H = \{H_1, H_2\}$

According to the first version of this decision problem to be considered, the credal state *B* recognizes all probability assignments of x to H_1 and 1-x to H_2 to be permissible where x takes values between 0 and 1.

The cross product rule and the expected utility rule together generate a set *Exp*(S) of permissible expected utility functions. In effect these are all probability weighted averages of the utility functions (55, - 45,), (-45, 55,) and (0,0).

The principle of expected utility claims that *Exp*(S) should be the value structure *V*(S) for *S*.

What remains to be explained is how the value structure is to be used in determining recommendations for choice. The remainder of this paper will be devoted to this topic.

2 Strict Bayesianism and Imprecise Probability

Strict Bayesians insist that if agent X is ideally rational, X's credal state for H_1 and H_2 should not consist of all distributions assigning values for x to H_1 in the unit interval and corresponding values 1-x to H_2. Nor should X adopt some convex subset of this set of distributions as X's credal state unless the subset is a unit set selecting one value for x as uniquely permissible. If that were done, the cross product and expected utility rules would then yield an expected utility function unique up to a positive affine transformation. The agent X's would be weakly ordered by this function with respect to better or worse and a set of optimal options could be identified. X could then maximize expected utility.

But even strict Bayesians can admit that circumstances are often less than ideal. X might be committed to adopting a uniquely permissible value for x but not be in a position to identify what that value is. X may be able to report a range of *possible* values for the uniquely *permissible* x. How is X to choose in the three way choice?

Sometimes X may find it helpful to consult a Bayesian statistician or decision theorist who may elicit from X information about X's uniquely permissible probability or expected utility function is. But such efforts at elicitation may not succeed in a timely fashion.

Perhaps, no option can be recommended. X remains in the dark as to what X's uniquely permissible probability is. Nonetheless, given that X is committed to having exactly one permissible probability in X's credal state, the following can be said: X ought not to choose *R*.

The reason is that *R* cannot be best in expectation in a three way choice when a single value of x is permissible. *R* can be better than G_1. If it is, however, G_2 must be better than *R*. Similarly, if *R* is better than G_2, G_1 is better than *R*.

There is yet another salient feature of strict Bayesianism to consider. According to the strict Bayesian, one should be prepared to make decisions both in the three way choice and in all the two way

choices as if one had recognized a definite value for x to be uniquely permissible. This is so even if one had not settled in advance that a definite value for x is uniquely permissible. In our example, one should maximize expected utility according to the same expectation function if the choice is between the three options or any pair of them. Indeed, one should be prepared to choose using the same value of x for determining expectations no matter which (finite) subset S of the set of all mixtures of $\{G_1, G_2, R\}$ constitutes the set of available options.

Thus, a strict Bayesian can admit that his state of credal probability judgment is *imprecise*. He may not be in a position to determine the value of x up to more than one or two decimal places. Or he may not be able to judge whether H_1 is more or less probable than H_2 or whether the two probabilities both equal 0.5. But when faced with a choice he is committed to making choices in other decision problems that cohere with the choice initially made in the sense that the choices made can all be represented as maximizing expected utility relative to the same probability.

3 Quasi Bayesianism and Indeterminate Probability

Imprecise probability is not to be confused with indeterminate probability. A strict Bayesian is committed to endorsing a determinate assessment of probability. Imprecision arises because the strict Bayesian may not be in a position to identify his commitment.

Quasi Bayesians allow that states of credal probability judgment may be indeterminate. Rational agents may recognize more than one probability function to be permissible in B. Such indeterminacy in probability judgment reflects the refusal of quasi Bayesian decision-maker Z to recognize exactly on probability distribution over the states to be permissible for use in deriving permissible expectation functions. Unlike the strict Bayesian X who may not know X's own mind, the quasi Bayesian Z may be very clear that Z recognizes more than one probability distribution to be permissible.

In all decision problems consisting of a choice among options belonging to a subset S of $\mathbf{M}(\Omega)$, the quasi Bayesian decision maker Z is restricted to recognizing as permissible evaluations in $V(S)$ just those derivable according to the cross product and expected utility rules via the expected utility principle from the permissible probabilities in B and the permissible utilities in $EV(O)$.

If a probability distribution is permissible according to Z, then any expected utility function that is derivable in the manner just indicated is permissible. If an option in S is optimal according to that expected utility function, it is an option Z has failed to eliminate from consideration as a candidate for implementation. The option is admissible for choice as far as considerations of expected utility are concerned. These considerations motivate the following definitions.

An option in S is *V-admissible* if and only if it is best according to at least one permissible valuation in $V(S)$. According to the expected utility principle, $V(S) = Exp(S)$. The options in S that maximize expected utility according to at least one expected utility function in $Exp(S)$ are called *E-admissible*. The principle of expected utility requires that the V-admissible options coincide with the E-admissible options. A quasi Bayesian insists that all options admissible for choice should be *E*-admissible.

When the available options consist of the three lotteries specified, Z can optimize according to at least one permissible probability by choosing G_1. He can optimize according to other permissible probabilities by choosing G_2. There is no permissible probability according to which R is optimal. Therefore, in the three way choice, R is not E-admissible but the other options are.

Thus far, there is little difference between Z and the strict Bayesian X. X agrees that R is not optimal in the three way choice. But consider the choice between G_1 and R. In this case, both options are E-admissible. And G_2 and R are E-admissible in the other pairwise choice.

The strict Bayesian X cannot abide this. X might rank G_1 and R together in the pairwise choice. But in that case, G_2 is uniquely optimal in the three way choice. And X might rank G_2 and R together in a binary choice. But G_1 must then be uniquely optimal in the three way choice. If, in the three way choice, G_1 and G_2 are both optimal, then R must be inferior on both binary choices where it is an option.

The choice consistency property γ stipulates that if an option is admissible for choice in every set S belonging to class of sets M, then that option is admissible for choice when the available options are those in the union of the elements of M. ([14], p.50.) If all the available options are weakly ordered as strict Bayesians require, property γ must reflect the way the decision-maker makes choices.

Choice functions defined by E-admissibility lack property γ. This makes sense as long as we are dealing with indeterminate probabilities. Choosing one of the E-admissible options among a set S of available options incurs no commitment to judge that option to be optimal according to the uniquely permissible expectation function as strict Bayesians require. It incurs no commitment to judge the option optimal according to all permissible expectation functions. The requirement is that the option be judged optimal according to one of the permissible probability distributions but not necessarily according to all. Z's choice behavior is not a symptom of irrationality according to quasi Bayesians but of the fact that some options are not comparable because expectations and probabilities are indeterminate.[1]

The idea of indeterminacy in probability judgment agrees well with the views of many of the older critics of what is now called the Bayesian view. I allude to authors like Venn and Peirce who maintained that unless one could derive subjective probability judgments from knowledge of objective statistical probabilities, one should refrain from using probability judgments at all. Instead of saying that probability is undefined in such cases, quasi-Bayesians say that the decision-maker may not be in a position to rule out all but one probability for use in determining expected utilities. The deliberating agent ought to recognize all those probability distributions that are not eliminated as permissible to use in determining expected utilities.[2]

4 Maximalism.

Bayesians and quasi Bayesians agree that no option should be admissible for choice unless it maximizes expected utility against some permissible probability-utility pair. To relax this requirement seems to go beyond what a good card carrying Bayesian can tolerate. Here is one principled way of making decisions that crosses the line dividing Bayesians from non Bayesians.

The Principle of V-maximality: If S is a subset of $\mathbf{M}(\Omega)$, an option in S is V-maximal relative to S and $V[\mathbf{M}(\Omega)]$ if and only if there is no option in S that is better than it according to all permissible utility functions in $\mathbf{M}(S)$.

According to *maximalists* like H.Herzberger [14] and A.K.Sen [3], rational agents are allowed to choose any V-maximal option in S. When the principle of expected utility is satisfied so that $V(S) = exp(S)$, the set of V-Maximal options coincides with the set of *E-maximal* options. P.Walley [17] endorses E-maximality as necessary and sufficient for admissibility without requiring E-admissibility.[3]

Thus in the three way choice between G_1, G_2, and R, all three options come out E-maximal. In a pairwise choice between any two of them, both options are E-maximal. The method of choice satisfies property γ. This holds quite generally.

Neither E-admissibility nor E-maximality guarantees a weak ordering of all options according to preference revealed by choice. But E-maximality deviates from the requirements of weak ordering only by allowing for intransitivity of revealed equipreference. E-admissibility is far more radical in its recognition of failures of revealed preference to yield an ordering. (See footnote 1.)

In my judgment, the fact that E-maximality deviates from weak ordering less dramatically than E-admissibility is neither a virtue nor a vice. But it is symptomatic of a pair of defects of considerable seriousness.

E-admissibility and E-maximality differ in their treatments of the relevance of the utility information carried in the permissible expectation functions in $Exp(S)$ and in their regard for requirements of Bayesian rationality. The differences may be brought out by comparing the first version of our illustrative decision problem with the following second version.

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[1] A "choice function" C satisfies property α iff the following condition is met: Given any nonempty sets S and T such that $x \in S \subset T \subset \mathbf{M}(\Omega)$, $x \in C(T)$, then $x \in C(S)$. C satisfies β iff for every y and z in $C(S)$, $x \in C(T)$ iff $y \in C(T)$. x is revealed to be weakly preferred to y according to C iff $x \in C(\{x,y\})$. C weakly orders $\mathbf{M}(\Omega)$ by revealed preference if and only if C satisfies α and β. The strict preference defined on the basis of weak revealed preference is transitive but indifference is not iff δ is satisfied. δ obtains iff whenever $x,y \in C(S)$, $C(T) = \{x\}$. C is normal iff $C(S)$ is the set of most preferred options according to weak revealed preference according to C. This holds iff α and γ hold. Choice functions for E-admissibility satisfy α but none of the other conditions. E-maximality violates β but none of the others. (See [15] 49-53.)

[2] The distinction between imprecise and indeterminate probability does not coincide with the usage of "imprecise probability" that figures in the title of this conference. I do not mean to engage in terminological dispute. It is the distinction that matters. Still I see no reason to deviate from the terminological practices I have followed since the early 1970's.

[3] The term "maximal" derives from Sen [14], p.9 in the social choice setting and is used by Walley [17], p.161. I used "E-undominated" in [6] p.136 in this sense and spoke of "V-noninferiority" in [7], p.94.) H. Herzberger [3] calls maximal options optimal in the "liberal" sense.

According to the second version, the credal state for *H* is the same as before. The extended value structure *EV*(*O*) differs as the following matrix indicates.

Illustrative decision problem:

Second Version

	H_1	H_2
G_1	$45	-$55
G_2	-$55	$45
R	$0	$0

The credal state for the states is maximally indeterminate as before. Now it turns out, however, that in a three way choice all three options are *E*-admissible. In a pairwise choice between *R* and one of the gambles, both are *E*-admissible.

Thus, the second version of our decision problem receives a different system of solutions according to quasi Bayesians than does the first version.

According to maximalists, however, there is no relevant difference as far a choice is concerned between the first and the second versions. In a three way choice, all three options are *E*-maximal in both versions. In a two way choice between any pair, both options are *E*-maximal.

The sets of permissible expectation functions in *Exp*(*S*) in version 1 and in version 2 are different. And this is reflected in the differences in the assessments of *E*-admissibility. According to *E*-maximalists, however, we should pay attention exclusively to the pairwise comparisons of the options determined by *Exp*(*S*). According to this approach, any pair of these options is noncomparable both in the version 1 example and the version 2 example. Consequently, we should regard any weak ordering of the three options consonant with this assessment to be permissible in both versions.

Thus, the criterion of *E*-maximality is insensitive to differences in the extended value structures and expectation structures of the two versions of the decision problem to which *E*-admissibility is responsive.

Moreover, the insensitivity in question reflects an indifference to the requirements of Bayesian rationality. Quasi Bayesians resist the demands of strict Bayesians for weak order as a condition of rationality. They think that rational agents can be in doubt concerning their probability judgments in the sense that they fail to rule some probability distributions for use in assessing expected utility

while ruling in others. They insist, however, that when an agent is in doubt as to which probability distributions to use in computing expectations, all distributions that are not ruled out are permissible for use in computing expected utility. And they insist that no option be admissible for choice if it is not optimal according to some permissible probability distribution.

Appeal may be made here to an analogy with full belief. One should never judge a proposition true if it is ruled out as not possibly true. One should never judge an option admissible if there is no permissible expectation relative to which it is optimal.

In his magisterial book, Peter Walley cites an example similar to version 1 of the example I am using here. ([17] 165, 3.99.) He observes that if the set of options is convex (or more specifically is closed under probability mixtures), the *E*-admissible and *E*-maximal options coincide. (See [16], ch.3.5.) But if we insisted that all mixtures of available options be available, we could not have either binary or three way choice. Walley seems to think that option sets can be closed under mixtures and, thinks it reasonable to assume that they are. [17], 162. If he were right, the differences between the quasi Bayesians and maximalists would be reduced to matters of formulation.

My own view is that decision problems where all mixtures of simple options are available as options are the exception rather than the rule. In any case, if Walley were correct, no rational agent could face a choice between a pair of options. The entire basis of revealed preference theory would then be undercut.

There is another way to bring *E*-admissibility and *E*-maximality closer together alternative to focussing on option sets closed under probability mixtures. I have already mentioned it. One can ignore the utility information carried by the permissible utility functions in *V*(*S*) and focus on the *categorical preference* over elements of *S*. [7], ch. 6.5. x is categorically weakly preferred to y in such a set if and only if it is weakly preferred to y according to every weak ordering in the set. The categorical weak preference relation yields a quasi ordering of *S*. Both versions of the illustrative example yield the same categorical quasi ordering of the options. All three options are non-comparable.

Consider all weak orderings that are consistent extensions of that categorical preference. Obviously in the two versions, the set consists of all possible weak orderings of the three members of *S*. Call this *V**(*S*). The set of options that are optimal according

to at least one weak ordering in this set are *V**-admissible. All three options are *V**-admissible in both versions of the illustrative example.

The *V**-admissible options are best against those rankings that would be permissible if we took only categorical preference among the available options into account in determining the set of permissible rankings. The set of options that are *V**-admissible coincides with the set of options that are *E*-maximal relative to *S*. The achievement incurs a serious cost. The utility information in *V*(*S*) is ignored except insofar as it carries information about categorical preference among the available options.

Quasi Bayesians think that the differences in the utility information carried by the value structures for the two versions of the illustrative example is highly relevant. In the both version, we should consider the set of extensions of the categorical quasi ordering over **M**(*S*) *that obey the requirements of Von Neumann and Morgenstern*. Then it becomes apparent that the value structures according to the two versions are quite distinct. The insensitivity of maximalism to such differences in utility information argues against indiscriminate acceptance of its precepts.

5 Security and S-admissibility

It is open to a decision-maker who recognizes several options to be E-admissible to adopt a secondary criterion for picking among them. A familiar one is maximizing the minimum security level. Security level is an ambiguous notion. It can mean worst possible outcome. It can mean minimum expectation. And other interpretations are possible. To fix ideas I shall understand security level to be mininum expectation. Let us call an option that maximizes minimum expectation in this way *among the E-admissible options*, *S-admissible*. ([5], 1974, [6], ch.7 and [7], ch.7.) In the three way choice, the S-admissible options are G_1 and G_2. *R* does not qualify even though its security level is higher because it is not E-admissible in the three-way choice. In saying this, it is not claimed that *R* is inferior to the other options. There is no best option and no worst option with respect to expected value. Nor is it claimed that that G_1 is equipreferred to the other E-admissible option G_2. All three options are noncomparable with respect to expected value or with respect to efficacy in realizing the agent's goals.

In spite of this, in a two way choice between G_1 and *R*, *R* is uniquely S-admissible. Using security as a secondary criterion on top of E-admissibility yields choice functions that lack property α. (See footnote 1.)

The failure of α and other popular choice consistency requirements like β and γ is the product of the failure of expected utility assessment to yield an all-things-considered weak ordering of the options when probability assessments are indeterminate. There are many permissible weak orderings with respect to expected utility. The decision-maker's choice function does not reveal an all things considered weak ordering.

Note also that appealing to security in this setting is not a counsel of pessimism. By hypothesis, we have already exhausted the resources that full and probabilistic belief provide in determining the E-admissible options. Invoking security at that point is not changing an imprecise state of probability judgment into a more precise, determinate and pessimistic one. To think that way is to think like a strict Bayesian.

According to quasi Bayesians, the appeal to security is an appeal to a salient and, perhaps, desirable property of options in contexts where *E*-admissibility and, hence, considerations of expected utility cannot render a verdict. Appeal to security is not appeal to a further criterion *E*-admissibility thereby creating a strict Bayesian solution.

I am not urging that rational agents be obliged to restrict choices to S-admissible options. Other secondary criteria might be invoked instead. In any case, S-admissibility is not a single criterion but a family of criteria parameterized by the decision maker's concern with maximizing one kind of security rather than another. If secondary criteria are invoked at all, they are expressions of a secondary aspect of the goals and values of the decision-maker and do not reflect any judgments about the probabilities of the consequences of the various options.

Walley's attitude towards maximizing lower expectations resembles the one just sketched. The main difference is that S-admissible options are options that maximize lower expectations among the E-admissible options. According to Walley, one would maximize lower expectations among the E-maximal options.

Options that maximize lower expectations among all available options need not be E-admissible. They are, however, E-maximal. So maximizing lower expectations among the E-maximal options is

equivalent to maximizing lower expectations among all options.

In an important essay [1], Gärdenfors and Sahlin propose an account of rational choice that also allows for representing probability judgment by sets of probability distributions. Unlike the proposal I favor, however, they do not require that options from which the decision-maker is entitled to choose should be E-admissible. They suggest instead maximizing the lower expected utility ([1], 324). In both versions of our example, the recommendation they favor is *R* in the three way choice as well as in any of the pairwise choices. Lower expectations induce a weak ordering of the options. There is no violation of either property α or property β. They cite this as an advantage of their proposal as compared to mine.

The difference between the approaches of Walley and of Gärdenfors and Sahlin is that Gärdenfors and Sahlin require the use of maximin. Walley takes maximin to be one of several secondary criteria one might deploy if E-maximality fails to render a verdict. He does not recommend it over the other secondary criteria.

Walley worries that in situations such as those illustrated by our example, the decision-maker might maximin by choosing *R* in a pairwise choice between *R* and G_1 and also in a pairwise choice between *R* and G_2. If one has been offered both opportunities for choice, one ends up with nothing whereas if one had chosen G_1 in the first case and G_2 in the second one would have gained a positive benefit.

I am not clear as to why Walley worries about this. If the decision-maker anticipates facing both decision problems, he should consider them together and choose G_1 and G_2. If when reaching one decision, the decision-maker does not anticipate reaching the second, there is nothing for him to worry about. If he is uncertain, he can factor his uncertainty into his calculations.

I just noted that maximizing lower expectations among the E-admissible options allows for even more striking failures of choice consistency requirements than restricting choice to E-admissible options does. Maximizing lower expectations among E-maximal options enhances choice consistency. But the cost is an even more dramatic breach with Bayesian requirements. In the three way choice, R is mandated by the policy of maximining among the E-maximal options.

Consider then a choice between some mixture of G_1 and G_2 and R in version 1. There are mixtures for

which lower expectation is positive and, hence, favored over R. The mixed option clearly dominates R and is to be chosen over it according to Gärdenfors and Sahlin, Walley and myself. Gärdenfors and Sahlin, nonetheless, *mandate* R in a three-way choice between G_1, G_2 and R. Walley *allows* adoption of a secondary criterion that recommends R. My approach *forbids* choosing R in the three way choice.[4]

5 Conclusion:

I have argued that strict Bayesians can acknowledge imprecise probabilities and represent such imprecision by a set of probability distributions. Indeterminacy in probability judgment is also represented by a set of probability distributions. These two representations are interpreted quite differently. Key to the difference in interpretation is the way the sets of distributions are used in rational choice. Following up this line of thought, I suggest that Walley's representation of states of credal probability judgment in terms of sets of probabilities should be understood in terms of how he proposes to use the members of a credal state in decision making.

From this point of view, so I have argued, Walley's advocacy of *E*-maximality attenuates the role of probability judgment in decision making by suppressing the cardinal aspects of permissible expectation functions and ignoring the demands of Bayesianity. *E*-admissibility does more justice to these considerations.[5]

I do not mean to suggest that the account of indeterminacy I have been advocating is trouble free. The use of the cross product and expected utility rules together has been challenged in recent years for two reasons:

The combined use of these rules to yield *Exp(S)* is sensitive to the way spaces of possible consequences

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[4] Because Gärdenfors and Sahlin use security as if it were a uniquely permissible ranking mandated by the value structure *V(S)* it appears that Seidenfeld's objection against insisting on weak ordering of options combined with failure of the sure thing principle applies to them. [11] 1988.) I am not convinced Seidenfeld's argument applies to Walley's view where the primary evaluation is in terms of expectations and the appeal to security is not mandated.

[5] I should mention that in [6], 136, I allowed for use of *E*-maximality in certain special cases. Thanks to the benevolent criticism of Teddy Seidenfeld, I have long since withdrawn from that view. (See [7], n.3, pp.232-3.) I now insist that no matter what secondary criteria may be invoked, *E*-admissibility is necessary for admissibility of available options for choice.

of options are partitioned. Is such partition sensitivity defensible and, if so, how? [4],[12].

Let decision-maker X face a choice betweein apair of options (the set S). X recognizes the expected utility function derived from probability p_1 and utility u_1 to be permissible and also the expected utility function derived from p_2 and u_2 where $p_1 \neq p_2$ and $u_1 \neq u_2$ according to the expected utility rule. Let the categorical quasi ordering determined by $Exp(S)$ preserve exactly those preferences representing the agreements in expectations for the functions derived from these two pairs. Then the cross product rule cannot be satisfied.[13]. Extensions of this result to n probability-utility pairs for finite n are found in [2] and [10].

These two objections are serious and deserve careful consideration. I believe, however, that they can be addressed. I respond to them elsewhere. [8], ch.9 and [9].

Quasi Bayesians hold that the main defect in the strict Bayesian point of view derives from the reluctance to allow for indeterminacy in probability and utility judgment. Probability can be indeterminate as well as imprecise. They also insist on respect for the strict Bayesian insistence that the options recognized to be admissible in any given decision problem ought to be Bayes solutions by the lights of the decision maker's indeterminate probability and utility judgments. Indeterminacy in probability judgment is thus recognized to be relevant to decision making in ways that maximalists, maximiners and other anti Bayesians often deny.

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