A survey of some applications of the idea of ambiguity aversion in economics

Sujoy Mukerji
Department of Economics
University of Southampton
Southampton SO17 1BJ, UK.
e-mail: sm5@soton.ac.uk

Abstract

Subjective uncertainty is characterized by *ambiguity* if the decision maker has an imprecise knowledge of the probabilities of payoff relevant events. In such an instance, the decision maker's beliefs are better represented by a set of probability functions than by a unique probability function. An *ambiguity averse* decision maker adjusts his choice on the side of caution in response to his imprecise knowledge of the odds. This paper attempts a (selective) survey of some of the achievements of the research program which has analyzed important economic phenomena using a methodology that departs from the standard paradigm by explicitly allowing for ambiguity aversion. We specifically look at applications, and implications, of ambiguity aversion in four areas: design of bilateral economic contracts, the trade in financial contracts and financial markets, strategic decision making and finally, the political economy of voting.

Keywords. ambiguity aversion, uncertainty, Knightian uncertainty, non-additive probabilities, capacities, Choquet expectation, economic contracts, financial markets, voting, auctions, public goods.

1 Introduction

Savage's theory of subjective expected utility maximization (SEU) [21] is the received paradigm used for modeling decision-making under subjective uncertainty in economics. A main implication of SEU that a decision maker (DM) behaves as if her subjective assessment of likelihoods of uncertain events may be represented precisely by a unique probability distribution. However, experimental evidence ever since [6] has shown this to be a palpably untrue description of behavior under uncertainty. It is often the case that a DM's knowledge about the likelihood of contingent events is consistent with more than one probability distribution. If you were to ask someone about the likelihood of a given eventuality, the answer you typically hear is, "between x and y%", rather than a crisp, "z%". But, does how precisely she knows the relevant odds, influence the choice of the typical DM? It does: DMs choose relatively conservatively in situations where information about the odds is ambiguous in the sense that a relatively wide range of odds is consistent with her knowledge. As Ellsberg had observed, imprecise information affected his experimental subjects in a consistent fashion: most preferred to bet on events with unambiguous rather than ambiguous odds (including, Savage himself!). And, he reported, even when faced with the evidence that this was inconsistent with SEU, most stood their ground, "because it seems to them the sensible way to behave". People adjusting their decisions depending on how well they know the relevant odds and acting with greater wariness the more vague their knowledge, is a commonly observed attitude, and has been named *ambiguity aversion.*

While there is a vast literature on ambiguity aversion (see [1]), and indeed of the many other departures from SEU, that convincingly establishes their importance in laboratory settings, this work has had little impact on the way that economics is done. In large part this is because there have been so few demonstrations that economically important phenomena can be understood by using, and only using, models other than the standard one (SEU). The formidable recent advances in formulating a very 'workable' analytical framework for handling ambiguity aversion have availed us with a wonderful opportunity of obtaining such demonstrations. And indeed, recent times have seen a growing research program which has availed itself of this opportunity. This paper surveys of some of the achievements of this research program. We specifically look at applications, and implications, of ambiguity aversion in four areas: design of bilateral economic contracts, the trade in financial markets, strategic decision making and finally, outcomes of voting procedures.

The pioneering axiomatic foundations of the principal formal ideas of the most widely used models of ambiguity aversion were provided in contributions by [22] and [13]. While the technical details of the formal models of decision making under ambiguity aversion used in the various applications vary, the broad intuitive content may be stated as follows. Suppose an agent's subjective knowledge about the likelihood of contingent events is consistent with more than one probability distribution. And further that, what the agent knows does not inform him of a precise (second order) probability distribution over the set of 'possible' probabilities. We say then that the agent's beliefs about contingent events are characterized by *ambiguity*. If ambiguous[1], the agent's beliefs are captured not by a unique probability distribution in the standard Bayesian fashion but instead by a set of probabilities. Thus not only is the particular outcome of an act uncertain but *also* the expected payoff of the action, since the payoff may be measured with respect to more than one probability. An agent's ambiguity of belief about an event is said to be greater, the greater the difference between the maximum and minimum probability estimate of the event, consistent with the agent's knowledge. An *ambiguity averse* decision maker evaluates an act by the minimum expected value that may be associated with it: the decision rule is to compute all possible expected values for each action and then choose the act which has the best minimum expected outcome. The idea being, ceteris paribus, the more an act is affected adversely by ambiguity the less its appeal to the ambiguity averse decision maker.

More formally, suppose that the DM's domain of uncertainty is the finite state space $\Omega = \{\omega_i\}_{i=1}^{N}$. The DM chooses between acts whose payoffs are state contingent: e.g., a financial asset z, $z : \Omega \rightarrow \mathbb{R}$. The ambiguity averse DM's subjective belief is represented by a convex set of (standard, additive) probabilities, denoted $\mathcal{C}$. The *ambiguity* of the belief about an event E is measured by the expression $\max_{\mu\in\mathcal{C}} \mu(E) - \min_{\mu\in\mathcal{C}} \mu(E)$. Like in SEU, a *utility function* $u : \mathbb{R}_+ \rightarrow \mathbb{R}$, $u'(\cdot) \geq 0$,describes DM's attitude to risk and wealth. The DM evaluates *Choquet expected utility* of each act, and chooses the act with the highest evaluation. The Choquet expected utility of an act is simply the minimum of all possible 'standard' expected utility values obtained by measuring the contingent utilities possible from the act with respect to each of the additive probabilities in $\mathcal{C}$:

$$\mathbb{CE}u(z) = \min_{\mu\in\mathcal{C}} \left\{ \sum_{\omega_i\in\Omega} u(z(\omega_i))\,\mu(\omega_i) \right\}.$$

The Choquet expected utility of an act is just its standard expected utility calculated with respect to a 'minimizing probability' corresponding to this act. Hence, in the Choquet method, the DM's appraisal is not only informed by his knowledge of the odds but is also automatically adjusted downwards to the extent it may be affected by the imprecision of his knowledge[2]

2 The design of (bilateral) economic contracts

Typically, economic contracts involve arrangements about contingent events. As such, the relevant tradeoffs hinge crucially on the likelihoods of the relevant contingencies. Hence, it is a reasonable conjecture that the domain of contractual transactions is one area of economics that is significantly affected by agents' knowledge of the odds. Thus contractual relations, crucial to the organization of a modern economy, is a natural choice as a particular focus of the research on the principal economic effects of ambiguity aversion.

Economics studies mutually beneficial exchange between individuals. The real world is dynamic and most mutually beneficial exchange takes place over time. One party renders a good or service in the present in exchange for the promise by another to render some good or service in the future. The basic way of organizing these intertemporal exchanges is through the use of contracts. Roughly put, the contracts which are commonly traded in an economy fall into two categories: those which are used for organizing (co-ordinating) production activity over time and those which are used for transferring (redistributing)

[1] To preempt misunderstandings it is emphasized that the term "ambiguity" as used in this paper, refers purely to the fuzzy perception of the likelihood subjectively associated with an event (e.g., when asked about his subjective estimate of the probability of an event, the agent replies, "It is between 50 and 60%."). It **does not** refer to a lack of clarity in the description of contingent events and actions. Also note, some authors and researchers refer to ambiguity variously as "vagueness", "Knightian Uncertainty" or even simply as "uncertainty". As it is used in this paper, the word "uncertainty" is simply the defining characteristic of *any* environment where the consequence of at least one action is not known for certain.

[2] [11] and [18] point out how the DM's awareness that the precise implication of some contingencies is inevitably left unforeseen, may lead to beliefs that have non-additive representation. The papers explain the Choquet decision rule as a 'procedurally rational' agent's means of 'handicapping' the evaluation of an act to the extent the estimate of its 'expected performance' is adversely affected by his imprecise knowledge of the odds.

income over time and across contingencies. The first category includes all supply and delivery contracts between firms, between firms and government, as well as all labor contracts. The second category consists of financial contracts: insurance, bonds, equities, futures and options.

The way many crucial economic institutions have developed and function is often best understood by studying the salient features of contracts and contractual relations underpinning the particular institutions. Take for instance the first of the two categories of contracts described in the preceding paragraph, and the modern theory of the firm. Why firms exist, what productive processes and activities are typically integrated within the boundaries of a firm, is largely explained by the nature of incompleteness of supply and delivery contracts. A contract may be said to be incomplete if the contingent instructions included in the contract do not exhaust all possible contingencies. However, incompleteness of contracts has largely been a puzzle to the standard theory. This has prompted researchers in recent times to consider alternative paradigms in their search for appropriate theories of incompleteness: e.g., [19] showed that contractual incompleteness can be explained by ambiguity aversion. This finding in turn explains some widely observed 'realities' about the organization of firms that were previously difficult to come to terms with. The formal analysis in [19] basically involves a reconsideration of the canonical model of a vertical relationship (i.e., a relationship in which one firm's output is an input in the other firm's production activity) between two contracting firms under the assumption that the agents' *common* information about the contingent events is ambiguous and that the agents are ambiguity averse. Next, we review this exercise with a simple example.

Consider two risk neutral firms, B and S. B is an automobile manufacturer planning to introduce a new line of models. B wishes to purchase a consignment of car bodies (tailor-made for the new models) from S. The firms may sign a contract at some initial date 0 specifying the terms of trade of the sale at date 2; that is, whether trade takes place and at what price. The value of the consignment to the buyer, v, its (marginal) cost of production c, and hence, the tradeable surplus $v - c$, are contingent upon the state of nature realized in date 1. There are three possible contingencies ω_0, ω_b, ω_s, with corresponding tradeable surpluses s_0, s_b, s_s. After date 0 but before date 1, S invests in research for a die that will efficiently cast car bodies required for the new model while B invests effort to put together an appropriate marketing campaign for the new model. The investments affect the likelihood of realizing a particular state of nature. Each firm may choose between a low and a high level of investment effort. The investments are not contractible per se but the terms of trade specification may be made as contingency specific as required. In the case that the contract is incomplete and an 'unmentioned' event arises with sure potential for surplus it is commonly anticipated by the parties that trade will be negotiated *ex post* and the surplus split evenly. Consider the two possibilities X and Y: X) there is a longer list of reservations for the new model than for comparable makes and at a price higher than those for comparable makes; Y) the variable cost of production of car body is low. The state of the world ω_0 is characterized by the fact that both the statements are false. At ω_b, X is true but not Y; conversely, at ω_s, X is false but Y holds. Correspondingly suppose $s_0 < s_b = s_s$. The common belief about the likelihood of ω_b is at the margin affected (positively) more by B choosing the high investment effort over low effort than by S doing the same, while the opposite is true of ω_s. As is customary, we define a (first best) efficient investment profile as one that would be chosen if investment effort were verifiable and contractible.

Bear in mind the allowance of being able to write complete contingent contracts and the institutional setting of a vertical inter–firm relationship. As is formally argued in [19], given all this and that decision makers are SEU maximizers, the non-verifiability of investment will not impede efficiency. In our example, for instance, a contract which distinguishes the three contingencies and sets prices that rewards B sufficiently higher at ω_b than at other contingencies (and similarly rewards S at ω_s) will enforce the first best effort profile. The general conclusion is that if agents are SEU maximizers then an incomplete contract which implements an inefficient profile cannot be rationalized. Such a contract can never be the optimal because it will be possible to find a complete contract that dominates it (i.e., a contract that obtains higher *ex ante* payoffs for both parties). However, this conclusion is overturned if agents are ambiguity averse. The logic of this may be seen by re-evaluating the above example with the *sole* amendment that agents are ambiguity averse. To provide sufficient incentive to take the efficient investment the *ex post* payoffs in the contract have to treat the two firms asymmetrically at ω_b and ω_s; for B the payoff is higher at ω_b than at ω_s, while it is the other way around for S. This implies that the firms would, in effect, use *different* probability distributions to evaluate their expected payoffs. From the set of probabilities embodying the firms' symmetric information B measures its payoffs using a probability distribution that puts a relatively higher weight on ω_s than the distribution

S thinks prudent to check its payoff against. Consequently, the sum of the expected payoffs will fall short of the expected total surplus — there is a 'virtual loss' of the expected surplus. It follows that if this 'loss' is large enough the participation constraints will break, thereby making such a contract impossible. An incomplete contract, say the *null* contract (one that leaves all allocation of contingent surplus to *ex post* negotiation), is not similarly vulnerable to ambiguity aversion. Such a contract will lead to a proportionate division of surplus at each contingency, implying that each firm will use the same probability to evaluate its payoffs. Additivity of the standard expectation operator then ensures that no 'virtual loss' occurs. It will be shown that from all this it follows that there will be parametric configurations for which an incomplete contract even though only implementing an inefficient investment profile, is not dominated by any other contract. Under such circumstances the market transaction, if maintained, may justifiably be conducted with an 'inefficient' incomplete contract. The 'inefficiency' of the market transaction would also explain why it might be abandoned in favor of vertical integration.

Why might an explanation like the one given above be of interest? The final section of [19] discusses historic instances of vertical mergers and empirical regularities about supply contracts that are understandable on the basis of ambiguity aversion, but are not well explained by 'physical' transactions costs of writing contingent details into contracts. A recurrent claim among business people is that they integrate vertically because of uncertainty in input supply. This idea has always caused difficulties for economists (see, for instance [2]) who have been unable to rationalize it and have generally regarded it as misguided. The analysis in the present paper explains how the idea of ambiguity aversion provides one precise understanding of the link between uncertainty and vertical integration. Finally, at a more abstract level, a significant insight obtained is that even if there were no direct cost to conditioning contractual terms on 'finely described' events, one may well end up with only 'coarse' arrangements because the value of fine-tuning is not robust to the agents' misgivings that they have only a vague assessment of the likelihoods of the relevant 'fine' events. The understanding that how well the DM thinks he knows the relevant likelihoods explains what events are used to condition contractual instructions, is a novel contribution of the theory of ambiguity aversion to the debate about the foundations of incomplete contracts, and, the economic theory of contract design. The understanding is indeed novel since to an SEU maximizer the quality or accuracy of his belief does not matter.

3 Financial contracts and financial markets

In a pioneering contribution, [4] identified an important implication of the CEU model with regard to optimal financial decision making. The paper showed that, in a static model with one risky and one riskless asset, given ambiguous beliefs and ambiguity aversion, there will be a multiplicity of asset prices that support the optimal choice of a riskless portfolio, giving rise to what is commonly called a "bid-ask spread". The intuition behind this finding is explained in the following example.

Suppose a risk neutral investor is considering a transaction involving a unit of a financial asset z with contingent payoffs. Specifically, the investor is comparing the expected payoff from buying one unit of the asset to that from short selling one unit of the asset. The following table indicates the (non-additive) probability describing the common information about the uncertainty and the contingent payoffs:

Possible states	ω_L	ω_H
Non-additive probability ν	$\nu(\omega_L) = 0.3$	$\nu(\omega_H) = 0.4$
State contingent payoff to buying	1	3
State contingent payoff to selling	-1	-3

The expected payoff of buying an unit of z, let us call it the act z_b, $CE(z_b)$ is obtained by taking expectations w.r.t. the relevant minimizing probability in the core of ν. Notice, the payoff from the act z_b is lower at ω_L than at ω_H. Hence, the relevant minimizing probability when evaluating $\mathbb{CE}(z_b)$ is that probability in $\mathcal{C}$ that puts most weight on ω_L. Therefore,

$$\begin{aligned}\mathbb{CE}(z_b) &= \min_{\mu\in\mathcal{C}}\left\{\sum z_b(\omega_i)\,\mu(\omega_i)\right\}\\ &= 0.6\times 1 + 0.4\times 3 = 1.8\end{aligned}$$

On the other hand, the payoff from going short on an unit of z (the act z_s) is higher at ω_L than at ω_H. In other words, buying and selling are non-comonotonic acts. Hence, the relevant minimizing probability when evaluating $\mathbb{CE}(z_b)$ is that probability in $\mathcal{C}$ that puts most weight on ω_H. Thus,

$$\begin{aligned}\mathbb{CE}(z_s) &= \min_{\mu\in\mathcal{C}}\left\{\sum z_s(\omega_i)\,\mu(\omega_i)\right\}\\ &= 0.3\times(-1) + 0.7\times(-3) = -2.4\end{aligned}$$

An 'economic' interpretation would run as follows. Given the ambiguity in the investor's subjective assessment of the uncertainty, more than one probability is consistent with his knowledge. Being ambiguity averse, he 'shades' the valuation to the extent it may be affected by the ambiguity. The switch in the relevant minimizing probability implicit in the evaluation when comparing a buying position to a selling, is simply a reflection of the 'shading effect'.

It is evident from our computations that if the price of the asset z were to lie in the open interval $(1.8, 2.4)$, then the investor would strictly prefer a zero position to either going short or buying. Unlike in the case of unambiguous beliefs (i.e., SEU) there is no single price at which to switch from buying to selling. Taking a zero position on the risky asset has the unique advantage that its evaluation is not affected by ambiguity. Thus price has to rise (fall) sufficiently to allow the investor feel secure in going short (long) by meeting the test of his conservative estimate–'shading' of valuations due to ambiguity aversion is what results in the 'inertia' zone. It is however important to note that Dow and Werlang's demonstration is simply a statement about optimal portfolio choice corresponding to *exogenously* given prices. Their result is not a description of an equilibrium since the model is not closed to obtain asset prices endogenously. [7] extend Dow and Werlang's analysis to an infinite-horizon, multiple-asset framework and find that the non-uniqueness of supporting prices is not restricted to riskless positions. More specifically, they generalize the [17] asset pricing model by allowing for ambiguous beliefs and ambiguity aversion and find that the non-uniqueness of supporting prices in general extends to the case where "there exist state variables affecting dividends that do not influence consumption". Since in this model asset prices may be indeterminate while consumption is exogenously specified, the paper provides a new 'explanation' for price volatility in asset markets, i.e., it can 'explain' the greater volatility for prices than for consumption that is observed in reality.

A financial contract is a claim to an income stream—hence the logic of the financial markets: by exchanging such claims agents change the shapes of their income streams, obtaining a more even consumption across time and the uncertain contingencies. A financial market is said to be complete if contingent payoffs from the different marketed securities are varied enough to span all the contingencies. However, in just about every financial market in the real world the span is less than the full set of contingencies, i.e., the markets are incomplete. The primary implication of incompleteness is that agents may transfer income only across a limited set of contingencies and are thus left exposed to risk in a suboptimal manner. Incompleteness of financial markets is a compelling feature because it explains crucial facts about the working of financial and competitive markets that would impossible to explain without assuming incompleteness. Indeed, this characteristic is the fundamental inspiration for the most comprehensive model of the market economy: general equilibrium with incomplete markets (GEI). Nevertheless, relatively little has been accomplished in the way of formally establishing of what precisely lead to the incompleteness. As one of the pre-eminent contributors to the literature on GEI , [10] comments: "Perhaps the most unexplored part of the GEI model is a theory explaining which markets are open and which are closed. This may be viewed as a challenge for a research program ... . Once a GEI model with endogenous asset formulation is developed... we could speculate about the conditions in which socially more important assets tended to be marketed before socially less important ones." The challenge then, is to obtain a model that yield conditions on primitives explaining why certain assets would not be traded even if they were available and could easily be developed.

[20] applies ambiguity aversion to provide an explanation of the incompleteness of financial markets. More particularly, the paper focuses on the question, "What prevents the typical bond-equity finance economy from offering sufficient opportunities for Pareto optimal risk sharing? In other words, why should the theorems of general equilibrium with incomplete markets (GEI), rather than general equilibrium with complete markets (GE), be a more compelling description of the typical bond-equity economy?" To analyze the question, the paper considers a stylized bond-equity economy, which though incomplete per se, has a rich enough set of assets available for trade such that given standard assumptions about behavior under uncertainty, the equilibrium allocation would arbitrarily approximate a complete market (GE) allocation. It is shown, however, that given 'sufficient' ambiguity aversion, a certain subset of the available assets will not be actually traded in equilibrium, even though available. Hence it is proved that, given 'sufficient' ambiguity aversion, provided the non-traded securities are non-redundant, equilibrium allocation of the bond-equity economy is a GEI equilibrium. This shows how ambiguity aversion may endogenously limit the scope of risk sharing obtainable through the bonds/equities *actually traded* in a typical economy, and therefore, explain why the actual behavior of such an economy is better described by the GEI model, rather than the GE model.

The underlying objective of the formal analysis in [20] is to identify the class of assets whose trade is vulnerable to ambiguity aversion: assets that will be traded if agents are subjective expected utility maximizers but not if the agents' common beliefs about payoffs of the assets is sufficiently ambiguous and the agents are ambiguity averse. It is found that what determines an asset's vulnerability to ambiguity aversion is whether its payoffs have an idiosyncratic component, i.e., if at least some component of the payoff is independent of the realized endowment vector and of the payoff of any other asset as well. It turns out that if, (1) the range of variation of the payoff's idiosyncratic component is 'large' relative to the range of the variation of the component correlated with the endowment vector and, (2) the ambiguity of the agents' common belief about the idiosyncratic component is sufficiently high, then the asset will not be traded in any general equilibrium of the finance economy. Moreover, we also find that the effect of idiosyncracy cannot simply be 'washed away' by the standard techniques of diversification relying on the laws of large numbers, as it would be if the agents' beliefs were not ambiguous.

The analysis and results in [20] suggest that if the increase in uncertainty were sufficiently great then trade in a certain subset of the assets will thin out (in particular, trade in those corporate bonds and forward contracts on equities for which the ratio of the range of variation of the idiosyncratic component to the range of variation due to the economic shocks is greater). History of financial markets is replete with episodes of increase in uncertainty leading to a thinning out of trade (or even seizing up completely) *peculiarly* in assets such as high yield corporate bonds ('junk' bonds) and bonds issued in "emerging markets"(vis., Latin America, Eastern Europe and East Asia)[3]. It appears only natural to interpret these episodes as one of drastic increase in the *common* uncertainty faced by investors, rather than as an increase in the asymmetry of information. Also, it seems eminently demonstrable that the high risk bonds which appear to be so sensitive to attacks of uncertainty are precisely those bonds which have high idiosyncratic components in their payoffs. Thus the theory of ambiguity aversion provides an endogenously generated 'natural' explanation of why only this certain class of assets, and not all assets, will be affected by the increase in uncertainty. The explanation is also useful in providing a novel understanding of the role of certain institutions of financial contracting in facilitating the transaction of corporate bonds.

Financial economists have observed that even though barriers to international investment have fallen dramatically, investors continue to allocate only a very small fraction of their portfolio to foreign investments. Indeed, the allocation is much smaller than would be expected in the absence of barriers to international investment. This evidence constitutes what is generally referred to as the home-bias puzzle[4]. It is suggested that ambiguity aversion may well hold an important key to unlocking the puzzle. There are two factors which suggest that ambiguity aversion may be the reason which leads investors to favor domestic over foreign assets in their portfolios. First, an investor may well have less information about payoff prospects of foreign assets as compared to domestic assets. [23] provide some survey evidence consistent with this view. Less information, may well mean more ambiguous information. Secondly, foreign assets may typically have more relatively pronounced idiosyncratic components in relation to the endowments of the domestic investor.

4 Strategic decision making

In recent years *game theory,* the theory of strategic decision making, has come to be basic building block of economic theorizing. Naturally, economists working with the ideas of ambiguity aversion have increasingly sought to incorporate the ideas into game theoretic analysis. While the theoretical work involved in making a success of this marriage is far from complete, there has already appeared some very innovative work in applying the newly obtained theoretical framework to explain sundry economic phenomena. In this section we shall review two examples.

The first relates to the theory of auctions. As is widely acknowledged, analysis of auctions is perhaps the most 'public' face of game theory in economics. No less important is the fact that the principles of auction theory lie at the very heart of the theory of regulatory design, and more generally, mechanism design. Traditional analysis of auctions assumes that each bidder's beliefs about opponents' valuations are represented by a probability measure. [16] examines the consequences of relaxing this assumption, by allowing for ambiguous beliefs and ambiguity aversion, in the first and second price sealed bid auctions where all participants are risk neutral, each bidder's valuation of the good being sold is known only to himself and is independent of others' valuations. Under a fairly general parametric specification of the model it is shown that the first price auction will be (strictly) preferred to the second price auction, by the seller. The result is of substantial interest, at least in part,

[3] Consider, for instance, the widely reported recent paralysis afflicting the junk bond markets in the U.S. and in Europe in the aftermath of the Russian and East Asian crises. For related press reporting see, "US corporate bond market hit," *Financial Times,* 13 October, 1998 and "Virgin arm abandons junk bond issue" *Financial Times,* 29 October, 1998.

[4] See [8] and [15] for references, as well as [9], [3], and [24].

because the traditional (SEU) analysis asserts that, given risk-neutral an bidders, the seller should essentially be indifferent between the prospects of the two auction formats. A brief intuition of the result may be given as follows. First, recall that the essential feature of sealed bid auctions is that each bidder submits a single bid to the auctioneer and the bid is not revealed to other bidders. In the first price format, the bidder who submits the highest bid wins and pays the price he bid. In a second price auction, the bidder who submits the highest bid wins but he is only required to pay a price equal to the second highest bid. As is well known, in a second price auction, irrespective of one's beliefs about others' valuations and bidding strategies, it is a (weakly) dominant strategy to bid one's true valuation. Bidding higher than one's own valuation, in a second price auction, increases the probability of winning only when the consequence is that one ends up paying a price greater than his valuation. Similarly, bidding below one's valuation is of no use: it decreases one's chances of winning when winning is gainful and does not affect the price paid in any case. However, beliefs do affect the bidding strategy in the first price auction. When bidders are considering the optimal bid in the first price auction, ambiguity aversion makes them behave as if it is likely that their competitors have a high valuation, thereby leading to a more aggressive bidding strategy.

Our second example is a paper that examines how strategic motives for 'free riding' are affected by ambiguity aversion. Free riding/cheating as a strategic imperative has been a central question of analysis for game theory ever since its early days: it is not for nothing that the *prisoners' dilemma* is a paradigmatic example of game theory. A recent contribution, [5] establishes that the incorporation of ambiguity aversion alters the predictions of the game theoretic analysis in very significant ways and indeed some of the new predictions are much more in tune with observed empirics than those of the traditional theory. [5] analyze the issue of public good provision where players have ambiguous beliefs about the other players strategies with regard to contributions toward the provision of a public good. The gist of the idea can be obtained by considering a 2-player game where a player i has the utility function $u_i(x_1, x_2) = g(x_1, x_2) - x_i$. $g(x_1, x_2)$ is the utility derived from the provision of the public good produced/provided using the players' respective contributions x_1, x_2. The overall effect of ambiguity aversion is to "downwardly bias" a player $i's$ expectation of player j's (equilibrium) contribution. Suppose g is concave. Given the downward bias, concavity of g implies an increase in the marginal benefit under CEU, as compared to that under SEU, of $i's$ own contribution. Hence, the equilibrium profile of contribution under CEU will involve higher contributions as compared to SEU. Indeed, as the authors discuss, this is what is consistent with the various empirical and experimental studies.

5 Voting behavior

Analysis of voting behavior has been an important part of economic analysis for a long time. However, the recent upsurge of interest in models of political economy have resulted in a renewed emphasis on obtaining a fuller understanding of the working of electoral institutions. While being a focus of analysis has meant that lot has become clearer, a number of issues remain murky and ill understood. Prominent among the latter is the so called "roll off" phenomenon: selective abstention in multiple elections. Many elections in the U.S., for instance, involve multiple ballots: at one time in the voting booth, the voters are invited to cast their ballots on a 'major' election (say, choosing the President or the Governor) and also on a number of 'minor' elections (say, choosing their representative on the local school board). A stylized fact about such elections is that voters typically cast their ballot on the major election but abstain from voting on the minor elections. [12] shows that this may be due to the differences in quality and quantity of information that voters have about each election, if we were to interpret the differences in information as differences in the level of ambiguity of beliefs. The crucial presumption is that the voters have ambiguous beliefs about the policies favored by the candidates in the minor elections. As such, the choice of casting a (pivotal) vote in favor or against a candidate in a minor election is pretty much analogous to the choice between going short or long on a financial asset as seen in the example (in section 3) based on Dow and Werlang's paper. Like in the matter of portfolio choice, here too, we obtain a zone of 'inertia' which translates into abstention.

6 Conclusion

The research program surveyed aims to demonstrate how departing from and reformulating aspects of the paradigmatic notion of rationality, aspects that do not sit well with empirical findings or common intuition, can yield crucial insight, hitherto unobtainable, into the working of vital economic institutions. Incorporating aspects of 'bounded rationality' in economic analysis is widely regarded as the most promising, if not challenging, frontier for theoretical economics. If successful, the research program will create a lasting impact on this frontier and move it afield in a very useful way. It would be particularly useful in inspiring studies that demonstrate the economic significance of other (i.e., other than ambiguity aversion; see e.g., [14]) commonly observed departures from SEU.

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