PhD project:

Dealing with Imprecision in Data

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Motivation

Possible deficiencies of data often occuring in practice:

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2. rounded or heaped values,
3. interval observations or multiple categories,
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Plan and goal: Review and compare different theoretical approaches to modelling and analysing imprecise data in order to find suitable methods for statistical analyses in practice.

One Considered Situation: Socio-Economic Survey Data

There are some strategies to correct the effects of the data's deficiencies on the estimates of a statistical model, for example:

- Measurement or classification error models or
- Sheppard's correction for rounded values.

These methods imply assumptions about which kind of imprecision is present in the data and about the prevailing error or rounding processes.

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Questions:

- How to collect data such that the observations (precise or / and imprecise) are reliable?
- Which data analysis tools can deal with imprecise observations and how do they perform compared to correction methods?

Example: Income Distribution - 1/4

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  Loss of information: precise answers impossible, coarse information.

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Idea: Income question as a combination of open question with option of answering according to a randomly presented category scheme.

$\Rightarrow$ Less missings, reliable precise or imprecise observations with overlapping intervals.

Example: Income Distribution - 2/4

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   - "coarse mix" data set: 60% of the data are coarsened in using 3 different category schemes
   - "coarse same cat" data set: all observations are categorized with the same category scheme
   - "round mix" data set: 60% of the data are rounded according to 3 different rounding mechanisms

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Data analysis:

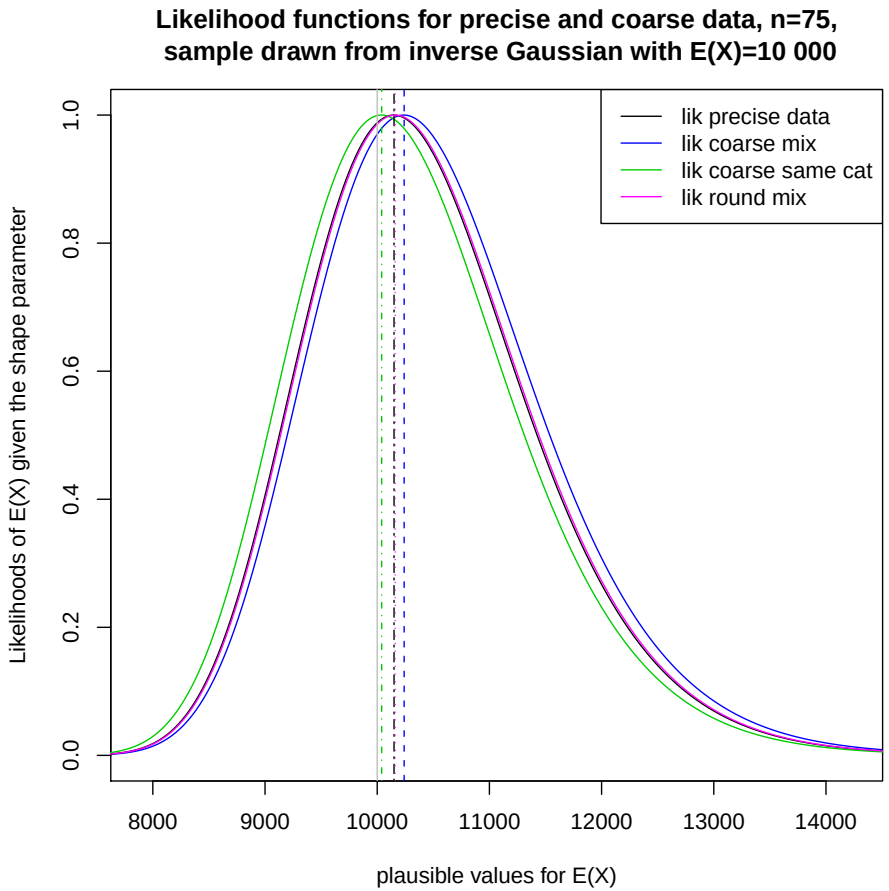
- Parametric method: Comparison of likelihoods
- Nonparametric method: Empirical distributions

Example: Income Distribution - 3/4

Parametric Inference:
Likelihood functions of $\mu = E(X)$ for different data sets with different kinds of imprecision.

$$L_\lambda(\mu, \mathbf{X}) = P_{invG(\mu,\lambda)}(\mathbf{X}),$$

$$\mu \in [X_{min}, X_{max}]$$



Example: Income Distribution - 4/4

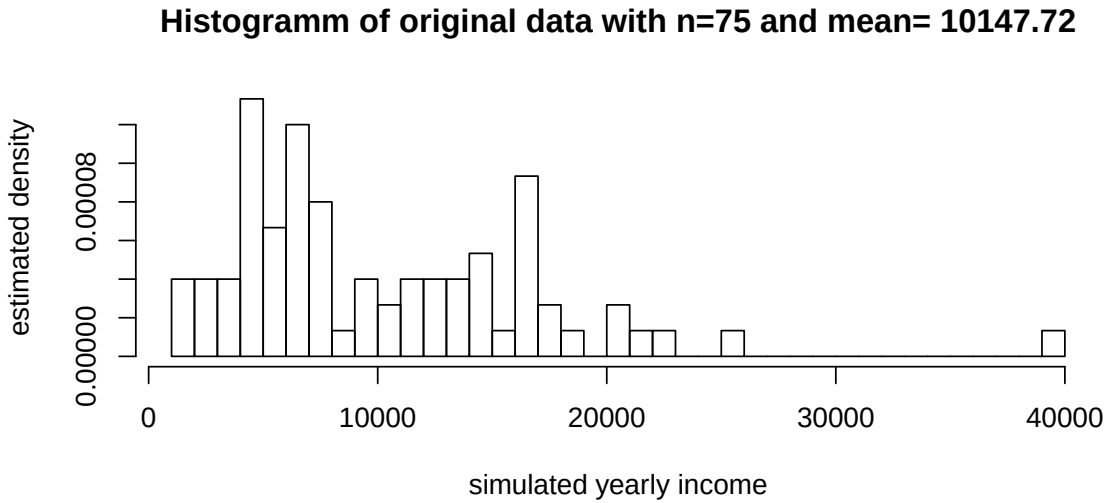
Nonparametric Inference: Empirical distribution.

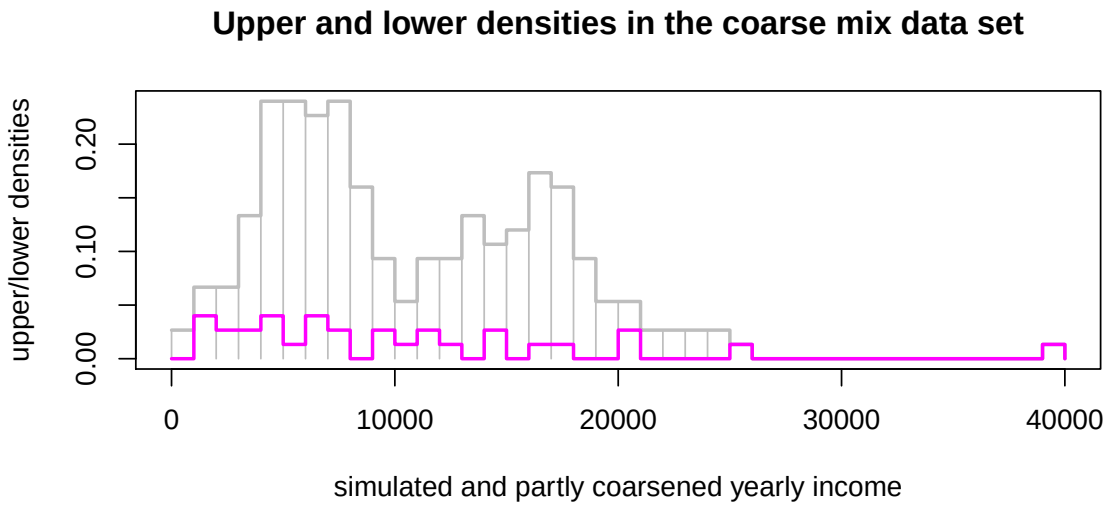
Upper/lower density of A:

$$\overline{f}(A) = \frac{1}{n}\sum_{i=1}^{n} I_{\{x_i \cap A \neq \emptyset\}}$$

$$\underline{f}(A) = \frac{1}{n}\sum_{i=1}^{n} I_{\{x_i \subseteq A\}},$$

A_j, $j \in \{1, \ldots, 40\}$ equal sized intervals, partition of $\mathcal{X} = [1, 40000]$.





Theoretical Concepts to be Considered

Conventional Statistics:

- Likelihood based inference
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Generalized Approaches:

- Random Set Theory
- Fuzzy Set Theory
- Imprecise Probability Theory