

# NONPARAMETRIC PREDICTIVE UTILITY INFERENCE

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## Which to choose?

Known fruits:



Newly discovered fruits:



(Dragon Fruit)

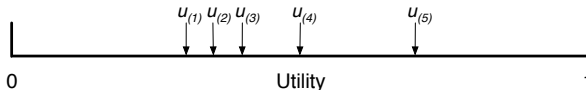


(Mangosteen)

## Which to choose?

Known fruits:

- Five previously experienced fruits  $f_1, \dots, f_5$  which, on a  $[0, 1]$  scale, have ordered utility values  $u_{(1)}, \dots, u_{(5)}$  equal to 0.3, 0.35, 0.4, 0.5 and 0.7:



Newly discovered fruits:

- Two alternative and unexperienced fruits  $f_{new}$  and  $f_{new2}$ .

What to select in a one off choice? What about a sequential choice?

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- This will facilitate the generation of a binary preference ranking  $\succeq$  determining preferences between any two implementable decisions, *i.e.*,  $d_i \succeq d_j$  denotes that decision  $d_i$  is at least as preferable as decision  $d_j$ .

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- In practice, however, the identification of a utility function with domain the set of available decisions is not necessarily straight forward.



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- Here we denote  $p_1 d_1 + \dots + p_n d_n$ , with  $p_i \geq 0$  as the mixed decision leading to outcome  $r_j$  with probability  $\sum_{i=1}^n p_i P_{r|d_i}(r_j)$ .

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- The collection of degenerate and mixed decisions then results in convex set  $\mathcal{D}$ .

## Review of Expected Utility Theory

The main result of von Neumann and Morgenstern (later generalised by others) is that, under the setting of the above, if agreement is accepted with a small number of axioms of rational choice, there exists a unique function  $u$  (up to positive linear transformation), with domain  $\mathcal{D}$  and co-domain  $\mathbb{R}$ , satisfying the following:

**P1:** For all  $d_i, d_j \in \mathcal{D}$ ,  $u(d_i) \geq u(d_j) \Leftrightarrow d_i \succeq d_j$ .

**P2:** For all  $d_i, d_j \in \mathcal{D}$  and any  $\alpha \in (0, 1)$ :

$$u(\alpha d_i + (1 - \alpha)d_j) = \alpha u(d_i) + (1 - \alpha)u(d_j)$$

P1 specifies that the utility function does rank decisions according to preference, whilst P2 indicates how utilities for mixed decisions are gained from utilities for degenerate decisions (or decision outcomes).

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- In reality people often learn their preferences by experimenting.
- This requires a generalization of the traditional concept of utility.
- Adaptive Utility, as first suggested by Cyert & DeGroot, is one such possibility.
- Basic idea rather simple: Treat utility in the same way that unknown random quantities are typically treated in standard Bayesian statistical inference, *i.e.*, subject them to a parametric belief model, and say that utility is only known up to the value of some unknown parameter  $\theta$ .

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  - How to elicit an appropriate likelihood linking the uncertain utility parameter with utility data?
  - What is appropriate utility data?

# Nonparametric Predictive Inference

Based on Hill's  $A_{(n)}$  assumption:

Let real-valued  $x_{(1)} < \dots < x_{(n)}$  be the ordered values of data  $x_1, \dots, x_n$ , and let  $X_i$  be the corresponding pre-data random quantities, then:

- 1 The observable random quantities  $X_1, \dots, X_n$  are exchangeable.
- 2 Ties have probability 0, so  $x_i \neq x_j$  for all  $i \neq j$ , almost surely.
- 3 Given data  $x_1, \dots, x_n$  and the definition that  $x_{(0)} = -\infty$ ,  $x_{(n+1)} = \infty$ ,  $I_j = (x_{(j-1)}, x_{(j)})$ , then for  $j = 1, \dots, n+1$ :

$$P(X_{n+1} \in I_j) = \frac{1}{n+1}$$

Note two random variables  $X$  and  $Y$  are exchangeable if

$P(X = x, Y = y) = P(X = y, Y = x)$ , and the concept formalizes the notion that the future is predictable on the basis of past experience.

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- Coincides with the general framework of a finitely additive prior and has been related to the theory of imprecise probability.
- Subjectivist interpretation of lower and upper bounds on betting price.

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- If utilities are scaled to  $[0, 1]$  how should the utilities for experienced outcomes be placed on that scale if we wish to allow the possibility that a novel outcome may be better (worse) than anything previously experienced?
- Here we suggest the interpretation that the utilities of experienced outcomes are placed on the  $[0, 1]$  scale by considering ‘hypothetical’ best and worst possible outcomes that could exist within the exchangeable taxonomic collection.

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- Let  $u_{(1)}, \dots, u_{(n)}$ , with  $u_{(i)} \in (0, 1)$  be the known ordered values of the utilities  $u_1, \dots, u_n$  representing preferences over outcomes  $\mathcal{O}_n = \{o_1, \dots, o_n\}$ .



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- Let  $\mathcal{U}_n = \{U_1, \dots, U_n\}$  denote the set of random quantities representing the utilities of the elements within  $\mathcal{O}_n$  before they are experienced, and suppose that the elements of  $\mathcal{U}_n$  are considered exchangeable.

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- Let  $\mathcal{U}_n = \{U_1, \dots, U_n\}$  denote the set of random quantities representing the utilities of the elements within  $\mathcal{O}_n$  before they are experienced, and suppose that the elements of  $\mathcal{U}_n$  are considered exchangeable.
- Given a new and novel outcome  $o_{new}$  whose utility value  $U_{new} \in (0, 1)$  is unknown but considered exchangeable with the elements of  $\mathcal{U}_n$ , the NPUI model considered here states only the following:

$$P\left(U_{new} \in (0, u_{(1)}]\right) = P\left(U_{new} \in [u_{(i)}, u_{(i+1)}]\right) = P\left(U_{new} \in [u_{(n)}, 1)\right) = \frac{1}{n+1}$$

## Expected Utility Bounds

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- Difference in utility bounds:

$$\Delta(E[U_{new}]) = \overline{E}[U_{new}] - \underline{E}[U_{new}] = \frac{1}{n+1}$$

# Updating

Expected utility bounds of a second novel outcome  $o_{new_2}$  once  $u_{new}$  is known:

- Lower updated expected utility bound:

$$\underline{E}[U_{new_2} | u_{new}] = \frac{n+1}{n+2} \underline{E}[U_{new}] + \frac{1}{n+2} u_{new}$$

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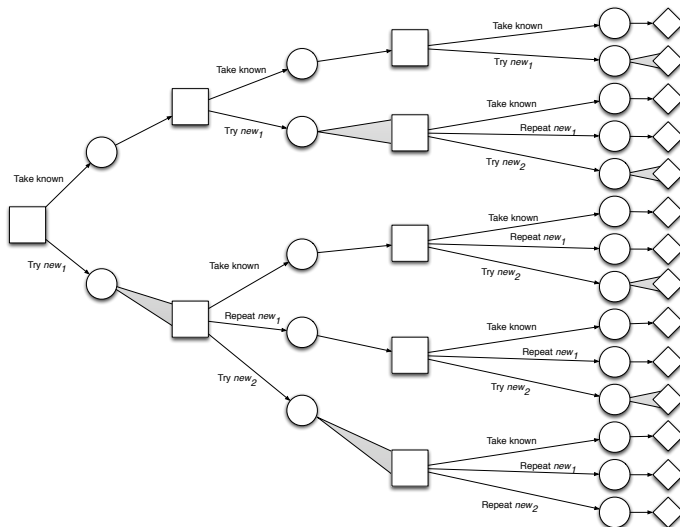
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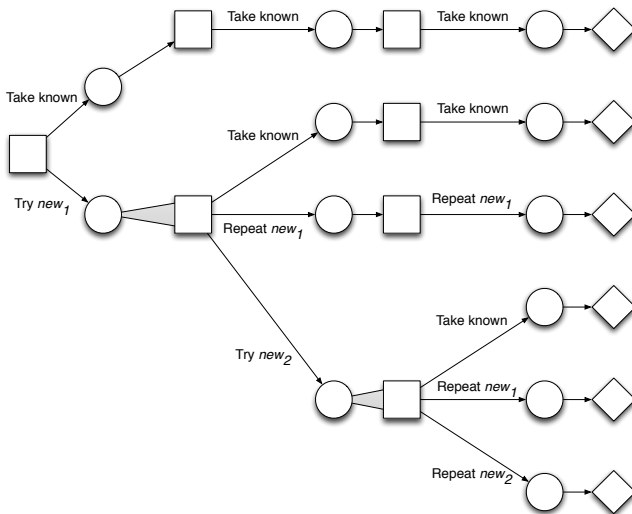
- Difference in updated utility bounds:

$$\Delta\left(E[U_{new_2}|u_{new}]\right) = \frac{1}{n+2}$$





# Reduced Decision Tree



## Sequential Choice Rules

In a sequential problem, a rule must be devised for choosing future decisions.

Extreme Pessimism:

The DM will always select the outcome or sequential decision path whose lower expected utility bound is greatest. Furthermore, future uncertain utility realisations will always fall at the infimum of any considered interval formed by the ordering of known utility values.

Extreme Optimism:

The DM will always select the outcome or sequential decision path whose upper expected utility bound is greatest. Furthermore, future uncertain utility realisations will always fall at the supremum of any considered interval formed by the ordering of known utility values.

## Conditioning

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- Lower conditional expected utility bound:

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- Internal Consistency:

$$\underline{E}[U_{new_2}] = \sum_{j=1}^{n+1} \underline{E}[U_{new_2} | U_{new} \in I_j] P(U_{new} \in I_j)$$

# Summary Results Table

Expected Utility for Optimal Decision Strategy

Available	Pessimistic		Optimistic		Select a Novel Option	
	Lower Bound	Upper Bound	Lower Bound	Upper Bound	Pessimistic	Optimistic
$f_1$	1.298	1.817	1.298	1.817	Yes	Yes
$f_2$	1.305	1.819	1.305	1.819	Yes	Yes
$f_3$	1.323	1.785	1.319	1.826	Yes	Yes
$f_4$	1.500	1.500	1.367	1.855	No	Yes
$f_5$	2.100	2.100	2.100	2.100	No	No

$i$	1	2	3	4	5
$u_{(i)}$	0.3	0.35	0.4	0.5	0.7

For the one-period problem:

$$E[U_{new}] = 0.375$$

$$\bar{E}[U_{new}] \approx 0.542$$



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- There has been limited discussion on the idea that preferences over decision outcomes may be uncertain, even though such scenarios have empirical support.
- How should uncertainty over preferences be incorporated within a normative decision analysis, and what are the implications of utility learning models?
- What sequential choice rule(s) should be employed?
- How to determine scaling within  $[0, 1]$  interval, or more generally, how to deal with the problem of induction when the actual value realized can be far better or far worse than anything as yet observed, and when it is the actual value that is important rather than the ordinal ranking.