Answers to exercises
Introduction to imprecise probability in environmental risk analysis

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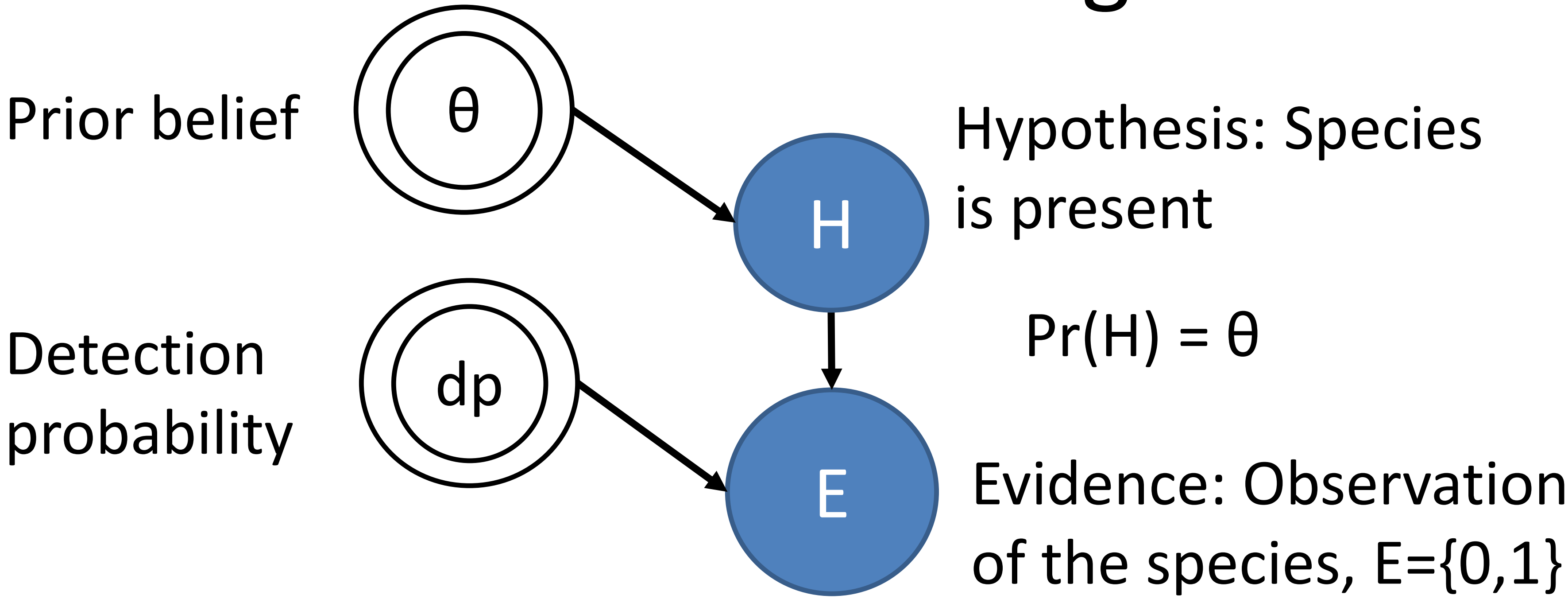
Partial knowledge



We did not observe the species, E = 0.

What is the probability that the species is still present?

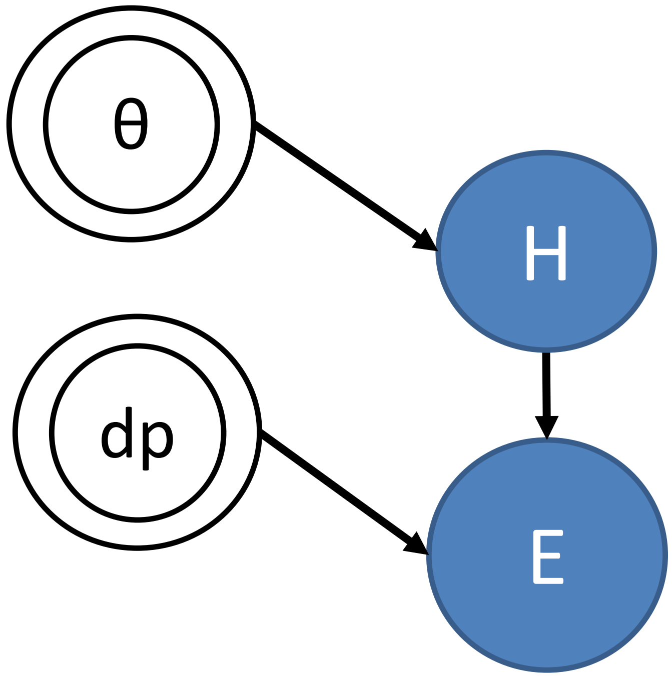
What to do when experts disagree on θ?

Quantify uncertainty in θ when dp is an interval?

Partial knowledge

Prior belief

Detection probability



Hypothesis: Species is present

Pr(H) = θ

Evidence: Observation of the species, E={0,1}

Pr(E = 1 | H) = dp

Pr(E = 1 | -H) = 0

$$P(H|E=0) = \frac{(1 - dp)\theta}{1 - dp\theta}$$

- What to do when experts disagree on θ?
  - Update Pr(H) for every expert's prior belief and bound it
- Quantify uncertainty in θ when dp is an interval?
  - Uncertainty in the data generating process

Daily intake exposure equation

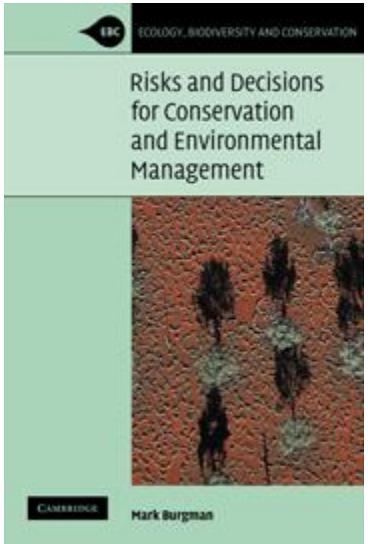
$$Dose = \frac{C\ x\ IR\ x\ EF}{bw}$$

C = concentration of chemial in medium (mg/l)

IR = intake/contact rate (l/day)

EF = exposure frequency (number of days year)

bw = body weight (mg)



Exposure data 1

C = [0.007, 3.30] x 10^{-3} mg/l

IR = [4, 6] l/day

EF = [45/365, 65/365]

bw = [4.514, 8.43] g

- What is the worst case exposure?
- Use Interval arthimetic!

$$\overline{Dose} = \frac{\bar{C} + \overline{IR} + \overline{EF}}{\underline{bw}}$$

Exposure data 2

$C = [0.007, 3.30] \times 10^{-3}$ mg/l

IR = [4, 6] l/day

EF ~ N([50,60] /365, 5)

- Quantify uncertainty in a high exposure to an organism with bw = 5?
- High exposure can be seen to occur in 1 day out of 100 (99th percentile).

Derive the lower and upper bound of the 99th percentiles based on the p-box for EF!

$$\overline{Dose} = \frac{\bar{C} + \overline{IR} + \overline{99th\ percentile\ for\ EF}}{\underline{bw}}$$

Exposure data 3

C = {0.001, 3.01, 0.74, 4.32, 2.9} x 10^{-3} mg/l
IR = {1.3, 4, 4.3, 5.9} l/day
EF ~ N([50,60] /365, 5)

- C, IR, EF varies over time (variability)
- Quantify uncertainty in a high exposure to an organism with bw = 5?
- High exposure can be seen to occur in 1 day out of 100 (99th percentile).

For example: Assume that data on C and IR are random samples from a distribution describing their variability.

A parametric approach would be to e.g. use truncated normal distriubtions for C and IR and learn about these parameters based on data. Since the sample sizes are small bounds on parmameters can be retrieved by using different sets of priors.

Propagate uncertainty using 2-dim MC or probability bounds analysis (it is enough to do a MC on the bounds of the C, R and EF parameters).

Exposure data 4

$C = [0.007, 3.30] \times 10^{-3}$ mg/l

IR = [4, 6] l/day

EF > 55/365

bw = [4.514, 8.43] g

- What is the worst case exposure?

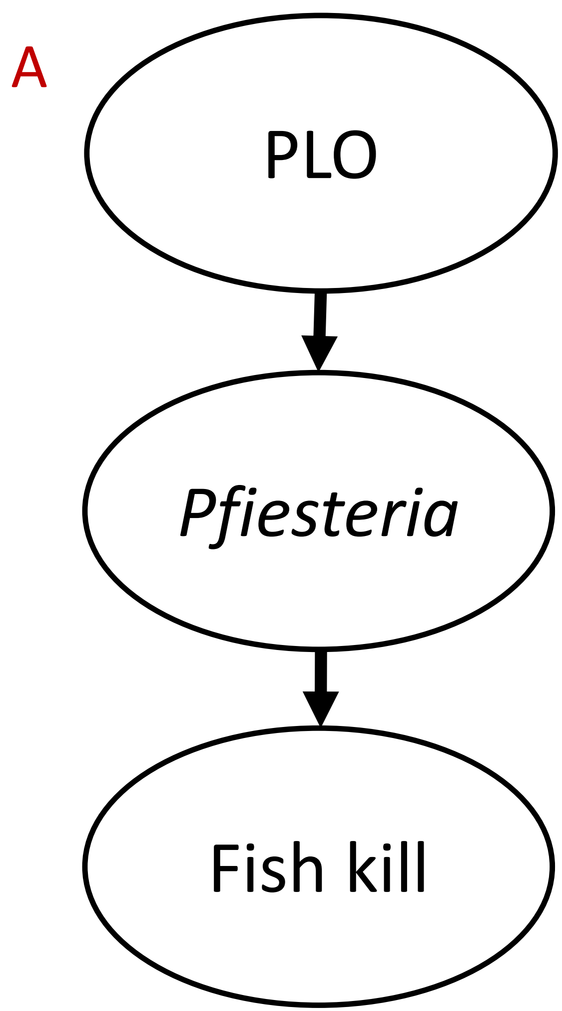
Well, there is acutally an upper bound on EF and that is 365/365.

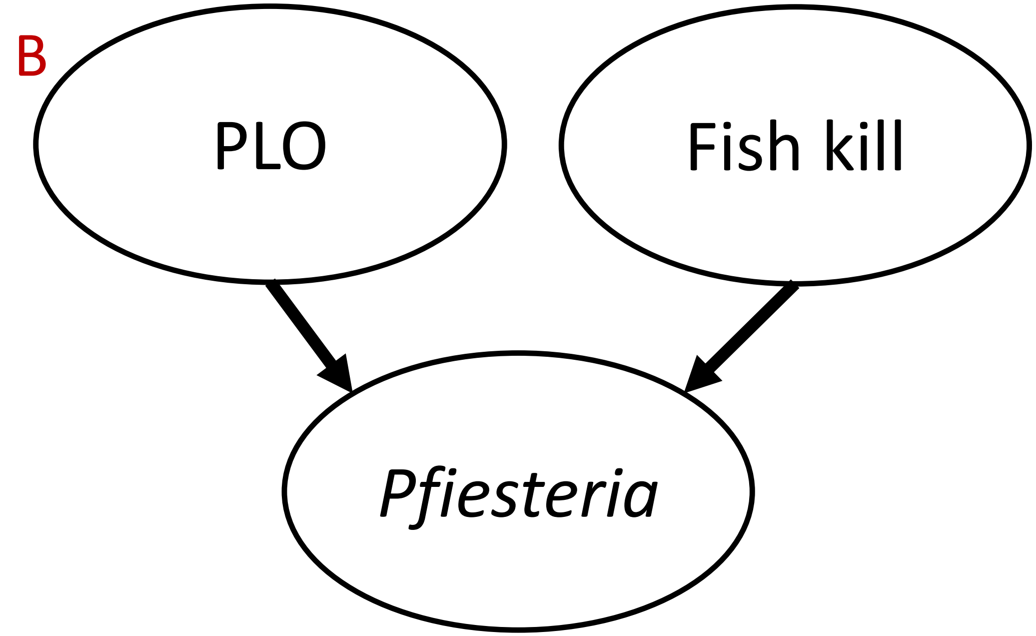
Then proceed and do worst case analysis as in data 1.

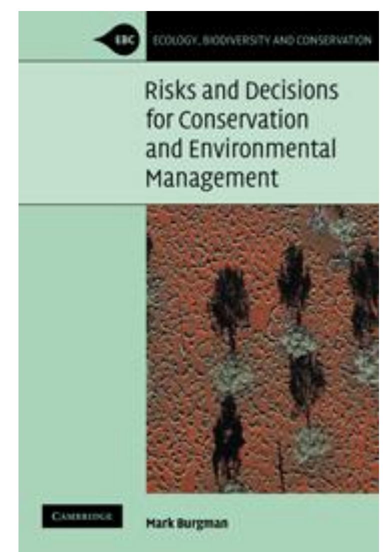
Causal model

- The purpose of this exercise is to do some calculations with Bayesian Belief Networks and understand why getting the causal sturcture accurate matters.
- PLO are the presencse of *Pfiesteria*-like organisms
- *Pfiesteria* is the presences of a toxic algae
- Fish kill is what it sounds like

Structural uncertainty







Structural uncertainty

- Pr(*Pfiesteria*) = 0.03
- Pr(PLO|*Pfiesteria*) = 1
- Pr(PLO) = 0.35
- Pr(Fish kill|Pfiesteria) = 1
- Pr(Fish kill) = 0.073
- Pr(*Pfiesteria*|Fish kill) = 0.38

Structural uncertainty

- What is the probability of Fish kills given that PLO is present under model A?
- Pfesteria is denoted by T (as in toxic algae bloom)

$$P(F|PLO) = P(F|T)P(T|PLO)$$

where $P(F|T) = 1$ and

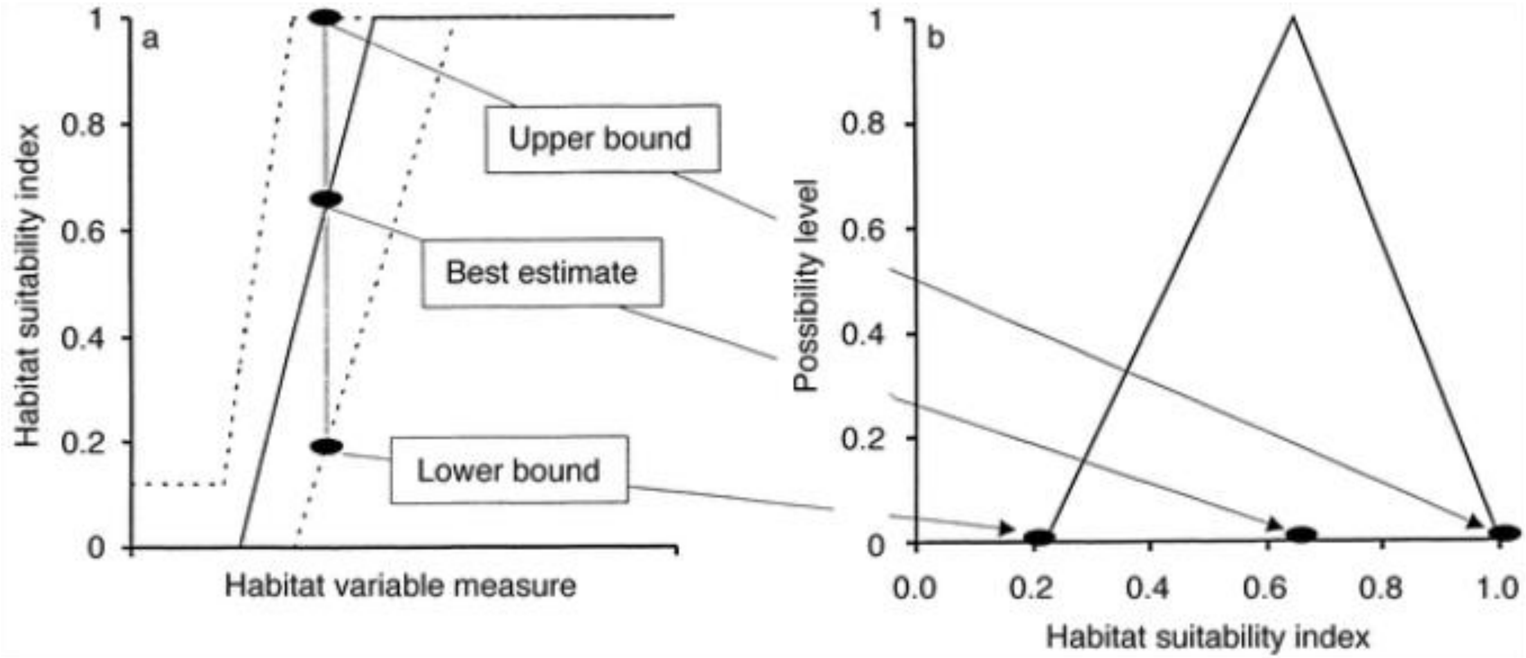
$$P(T|PLO) = \frac{P(PLO|T)P(T)}{P(PLO)} = \frac{1 \cdot 0.03}{0.35} = 0.09$$

Thus $P(F|PLO) = 1 \cdot 0.09$

Structural uncertainty

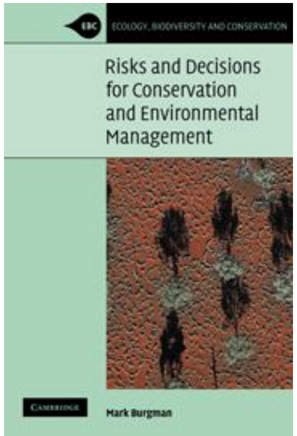
- *Pfiesteria* were only present at fish kill sites and never elsewhere.
- Therefore the assessors propose that model B is more accurate
- What is the probablity of Fish kills given the PLO is present under model B?
- $P(F|PLO) = P(F) = 0.073$ since Fish kill and PLO are independent and we do not know the state of their common child node

A prioritization problem

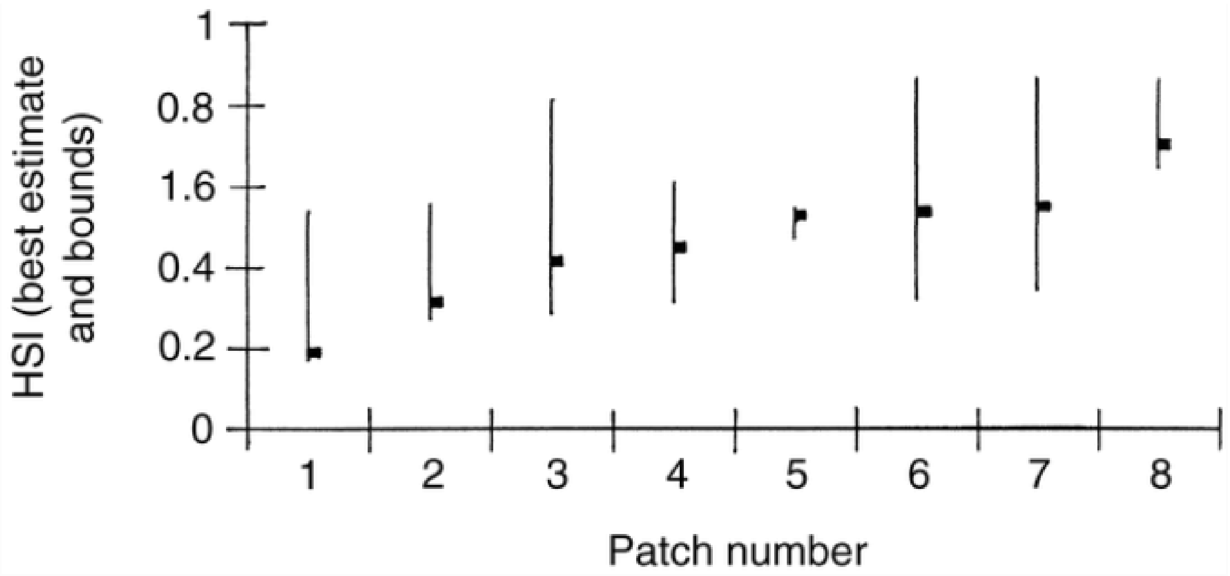


SETTING RELIABILITY BOUNDS ON HABITAT SUITABILITY INDICES

Ecological Applications
Volume 11, Issue 1, pages 70-78, 1 FEB 2001 DOI: 10.1890/1051-0761(2001)011[0070:SRBOHS]2.0.CO;2
http://onlinelibrary.wiley.com/doi/10.1890/1051-0761(2001)011[0070:SRBOHS]2.0.CO;2/full#i1051-0761-11-1-70-f01



A prioritization problem



- Which patch should be prioritized for conservation? Patch 8 if we want to maximise the lower bound.
- What if we need to eliminate a patch, which one should we take? Patch 5 if we want to minimize the upper bound and be sure we do not loss any good habitat

Spatial planning using PVA

- Two nature reserves d distance apart
- $1/\beta$ = mean disperal distance
- $U(\beta, u) = \left[(1-u)\tilde{\beta}, (1+u)\tilde{\beta}\right]$,

where $0 < u < 1$ and $\tilde{\beta}$ = 0.05 is the best guess

- q = the probability of persistence of the metapopulation under a long time horizon given by a meta-population model
- Optimal persistence when β is precise is

$$R(\beta) = \max_d q(d)$$

Spatial planning using PVA

- What distance should be between the reserves to make sure the persistence is acceptable, i.e.

$$\left[\min_{\beta \in U(\tilde{\beta},u)} R(\beta)\right] \geq Q$$

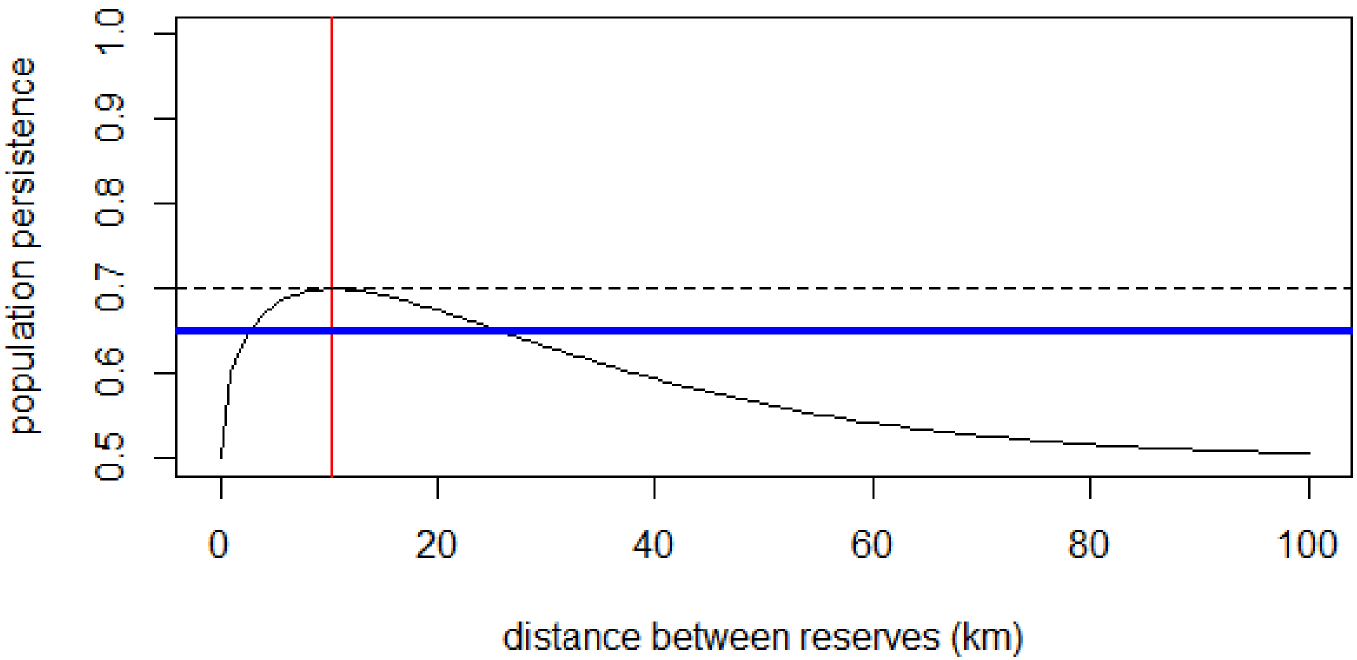
$$q = \frac{e^{-\beta \cdot d}(2 \cdot p_e - 1) - (p_e - 1)[2 + (e^{-\alpha \cdot d} - 1) \cdot p_e]}{2} + \frac{\sqrt{4 \cdot (p_e - 1)[(e^{-\beta \cdot d} + p_e - 1)(p_e - 1) - e^{-\alpha \cdot d} \cdot p_e(p_e - e^{-\beta \cdot d} - 1)] + [2 - 3 \cdot p_e - e^{-\alpha \cdot d} \cdot p_e(p_e - 1) + p_e^2 + e^{-\beta \cdot d}(2 \cdot p_e - 1)]^2}}{2}$$

This function is in the file reservedesign.R

Halpern, B. S., Regan, H. M., Possingham, H. P., & McCarthy, M. A. (2006). Accounting for uncertainty in marine reserve design. Ecology Letters, 9, 2-11.

Spatial planning using PVA

find_opt_and_plot(beta=0.05,pc=0.5)

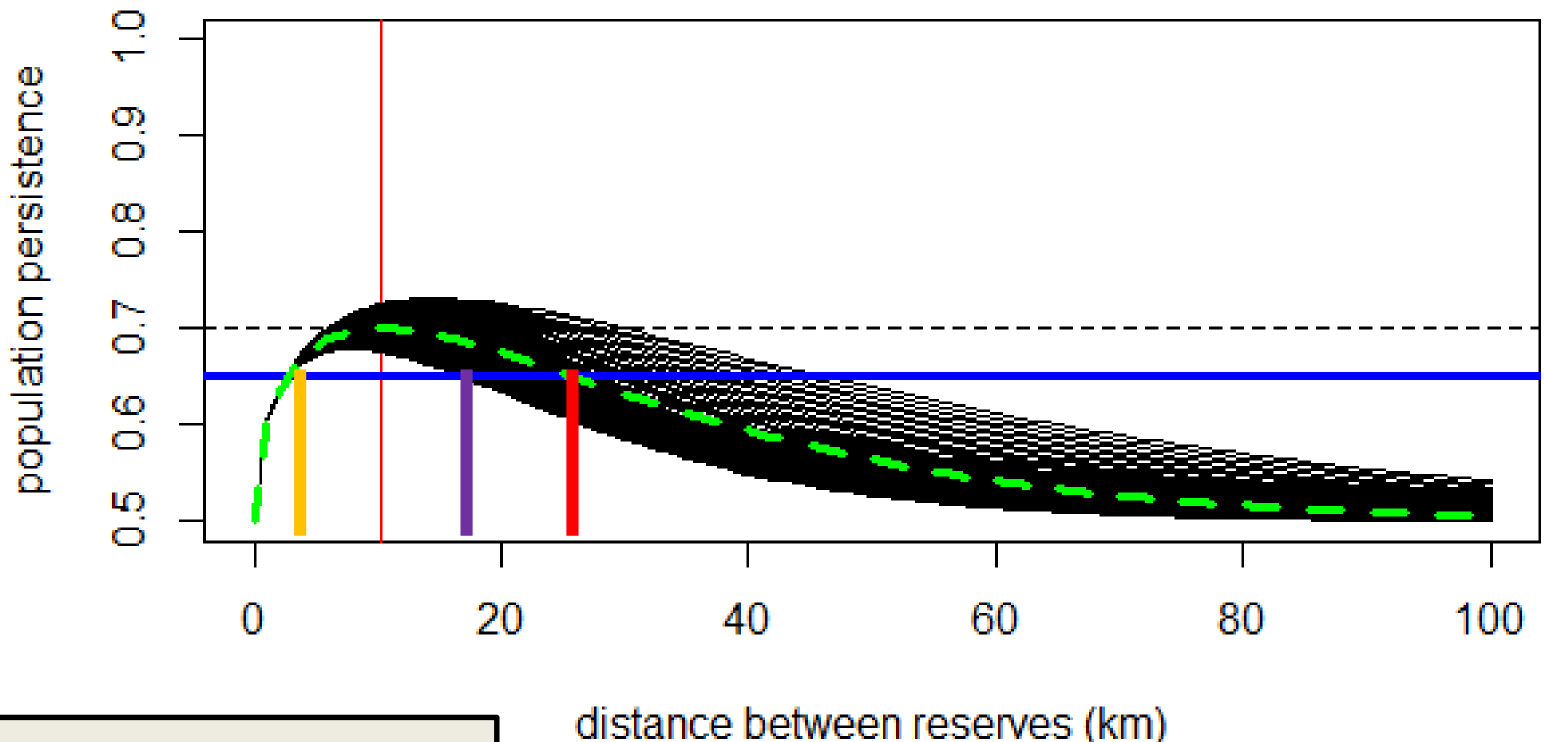


If Q = 0.65, there is a range of distances that could lead to an acceptable population persistence

reservedesign.R

Spatial planning using PVA

persist_over_d_unc(u_plus=0.4,beta_tilde = 0.05,pc = 0.5,color = 'black')



reservedesign.R

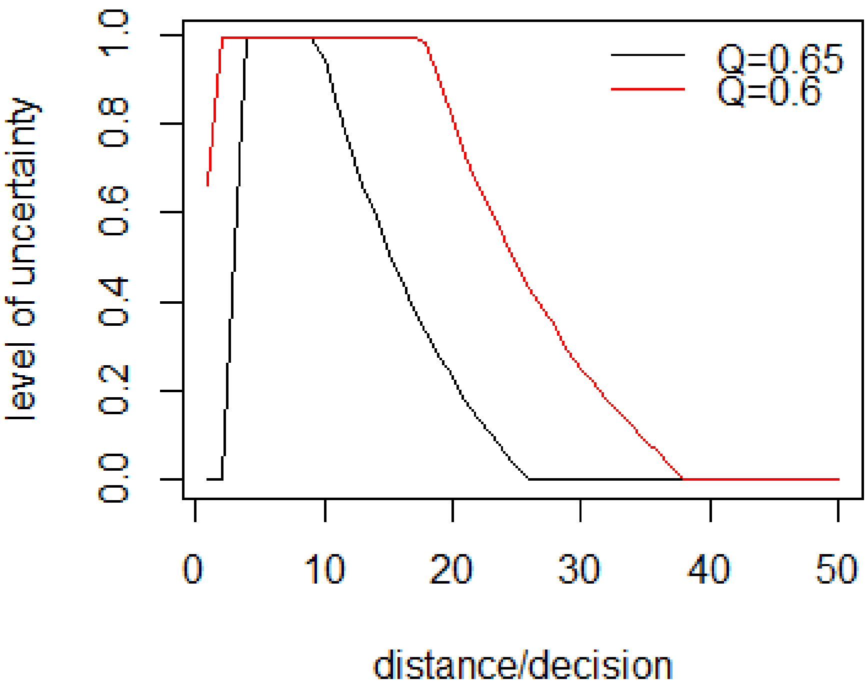
When we allow for imprecision the upper bound of acceptable distances changes from red to purple.

Info-gap analysis

- Find the distance d which allows the most uncertainty in $1/\beta$ (i.e. the mean disperal distance)
- $\hat{u}(d, Q) = max\left\{u: \left[\min_{\beta \in U(\tilde{\beta}, u)} R(\beta)\right] \geq Q\right\}$

Halpern, B. S., Regan, H. M., Possingham, H. P., & McCarthy, M. A. (2006). Accounting for uncertainty in marine reserve design. Ecology Letters, 9, 2-11.

Info-gap analysis



- Robustness under two criteria for what is an acceptable decision

reservedesign.R

u_hat = info_gap(Q = 0.65,d = d,beta_tilde = 0.05,pc = 0.5)